ANALYTICAL STUDY OF INITIALLY BENT STEEL PIPE PILE IN LAYERED SOILS

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By

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CERTIFICATE

This is to certify that this work "Analytical Study of Initially Bent Steel Pipe Pile in Layered Soils," by Mr. Kavuri Siva Rama Prasad, has been carried out under my supervision and it has not been submitted else where for a degree.

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(Kavuri Siva Rama Prasad)

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NOTATIONS AND SYMBOLS

a_0, a_1, a_2, a_3, a_4, a_5	: Constants
A	Area of Pile
A(I)	Recurrence Coefficients at node i
b_0, b_1, b_2, b_3	Constants
B(I)	Recurrence Coefficient at node i
C	Initial pile offset of bent pile
C_1, C_2, C_1', C_2'	Constants
C_u	Undrained shear strength
d	Internal diameter of steel pipe pile
D	External diameter of steel pipe pile
D_1, D_2, D_1', D_2'	Constants
E	Young's Modulus of steel.
F_1, F_2	Shear Force at top and bottom of pile
FA(I), FB(I) FC(I)	Coefficients at node i
h	Increment length
i	Node i
I	Moment of inertia
k	Coefficient of subgrade reaction
K	Soil Modulus or Subgrade modulus

K_z	Dimensionless depth at which initial shear force is zero
L	Length of the pile
L_1	Depth or thickness of top layer
m_1, m_1, m_2', m_2'	Constants
M_1, M_2	Initial and additional bending moments
n	Number of segments or exponent in the polynomial variation of soil modulus
n_{ht}, n_{hb}	The coefficients of modulus variation for the top and bottom layers
P	Load applied on the bent pile
P_{cr}	Critical buckling load
P_y	Load in the pile at any depth y
q_1, q_2	Initial and additional soil reactions on the bent pile
r	Radius of gyration
r_o	Non-dimensional soil modulus at the pile head in non-homogeneous soil
R	Radius of curvature of initial shape of the bent pile
R_o	Layer coefficient
RHS'(I)	Right hand side of differential equation at node i
S_1, S_2, S_3	Initial flexural stress, additional flexural stress and axial stress respectively

T	Relative stiffness of the pile
u, u_1, u_2	Total deflection, initial deflection and additional deflection respectively
u_{2t}, u_{2b}	Additional deflections in top and bottom layers respectively
w, w_1, w_2	Nondimensional total deflection, initial deflection and additional deflection respectively.
$(w_2)_i$	Dimensionless additional deflection at node i
w_{2t}, w_{2b}	Dimensionless additional deflections in top and bottom layers
W2(I)	Dimensionless additional deflection at node i
y	Depth along the pile
z	Non-dimensional depth coefficient
z_1	Non-dimensional thickness of top layer
z_m	Non-dimensional length of the pile
z(I)	Depth at node i
α	Non-dimensional load on the pile
α_c	Load carrying capacity of straight pile
φ	Skin friction factor for homogeneous or non-homogeneous soils
φ_1, φ_2	Skin friction factors at the top and bottom layers
β_1, β_2	Non-dimensional skin friction parameters
θ	Initial slope at the bottom of Fix-Pin pile
θ_1, θ_2	Initial slopes at the top and bottom ends of pin-pin pile

ABSTRACT

The observations of pile bending during driving have been reported as early as in 1930. Since then many investigators have discussed the bearing capacity of bent piles. Response of the initially bent pile to the externally applied load is one of the complex soil structure interaction problem. Review of literature indicates that no systematic analytical work has been done on initially bent piles in non-homogeneous soil which is commonly encountered in practice. An effort is made in this investigation to analyze the behaviour of initially bent steel pipe piles in layered soil using Winkler s hypothesis. It is assumed that the coefficient of subgrade reaction for each layer is constant and the ratio of the subgrade reactions of top layer to bottom layer is represented by layer coefficients. Polynomials satisfying both geometric and natural boundary conditions are selected to represent the initial shapes of the bent piles. Three end conditions, fixed-fixed, fixed-pinned, and pinned-pinned are considered in this analysis. The load carrying capacities of straight piles are calculated according to the AISC specification for the Design, Fabrication and Erection of Structural Steel for Buildings and the load

carrying capacity of the bent piles are expressed as a percentage load carrying capacity of the straight piles. The effect of initial shape on the load carrying capacity of bent piles is studied assuming number of polynomials for each set of end conditions. The effect of initial shape on the load carrying capacities of bent piles is shown to be most significant. The effect of layering on the capacities of bent piles is found to be negligibly small.

The influence of side friction on the capacities of initially bent piles is investigated for layered cohesive and cohesionless soils using finite difference technique and the method suggested by Glesser (1953). Down drag effects are investigated for cohesive soil. The effect of polynomial variation of subgrade reaction on the capacities of initially bent piles is also investigated. The increase in the capacities of bent piles due to soil friction is shown to be significant in case of cohesive soils especially for smaller pile offsets and longer lengths. The down drag effects are found to be severe in reducing the capacities of bent piles. The polynomial variation of soil modulus has a minor effect on the capacities of bent piles.

CHAPTER I

INTRODUCTION

Pile foundations are generally resorted to when either the subsoil is poor, the loads to be carried are heavy or the settlement becomes excessive. A Pile transmits the load from the structure to the soil either by point resistance or side friction or both. The design and construction of pile foundation is still an art and a challenge to the designer.

In designing the pile foundation it is generally assumed that the pile member beneath the ground is in perfect condition i.e. straight and free from initial bending stress. But perfection is rare and the pile foundations are no exception to this. Engineers associated with piling over the years have come to recognize the fact that not all the piles are driven straight, no matter how carefully it is driven. To account this very often driving tolerance such as less than 1 in 80 or similar recommendation is specified for construction. Awareness of the problem of piles not driven straight arose with the extensive use of steel piles in Norway in the 1930s. Since then, many cases have been reported as listed in Chapter II.

The classification of piles based on the initial shape is shown in Fig. 1.1. When a pile is smoothly and uniformly curved without any sharp kinks, it is called bent pile. The bending can be in one or more planes. A pile whose axis shows some deviation from the perfect straight line before loading is known as imperfect pile. A pile may have kink or develop a very sharp curvature locally instead of smooth and uniform curvature. Such a pile is called doglegged pile.

In practice, the reasons for piles not driving straight may be one or more of the following: (1) lack of straightness in the pile, (2) slight ecentricities of load application during driving, (3) asymmetric failure pattern in the soil at the pile tip, (4) the longitudinal and transverse vibrations imparted to the pile during driving, (5) the presence of adjacent piles, other obstructions and near by excavation, (6) variations in soil density, (7) slope of bed rock, (8) splicing of various segments of pile and (9) other considerations such as slackness in the guides as they determine the direction of entry of the pile into the ground.

The pile bending phenomena during driving has important practical implications if (a) piles are members of a group and a pile which has diviated affects the driving of other members of the group. (b) The structural integrity of the pile is impaired due to large deviation during insertion. (c) Piles designed to go through upper layers of soft soil and bear on firm

underlying strata due to large deviations. (d) Due to large deviations, the application of the working load produces additional deflections of the pile which are sufficient to cause yield of the soil, the carrying capacity of the pile might then be impaired.

The capacity of the pile is governed by one of the following considerations:

(i) The capacity of the soil to support the load transmitted by the pile.

(ii) The capacity of the pile to transmit load to the soil.

The former consideration is generally governed by the properties of the adjoining soil. The later consideration is known as structural capacity which depends on the strength of pile material and the pile cross-section. The pile material can fail either due to excessive buckling or excessive compressive stress. The axially loaded pile foundations are designed for the capacity of the soil to support the load transmitted by the pile divided by a suitable factor of safety. However, if the pile is bent the load carrying capacity of pile is dependent on the structural strength of the pile because the residual stresses developed in the pile due to initial bending constitute significant portion of allowable stress or yield stress for the pile material. For economy, the straight piles are designed such that both the pile material and soil are stressed to their maximum allowable value. If the economically designed straight piles get bent during driving, the load

that can be imposed on such a pile without overstressing the pile material is certainly less than the load carrying capacity of the soil. In the present investigation it is assumed that initial bending occurs in an economically designed pile which causes reduction in load carrying capacity of the straight pile.

The work described herein is highly idealized. This is deliberate because it is argued that until the general mechanics are fully appreciated and understood, there is little hope for improving design methods to enable the load carrying capacity of initially bent pile to be quantified or for improving construction techniques to enable piles to be driven straight. However, the computed behaviour should be qualitatively correct whereas quantitatively approximate.

The soil is represented by Winkler medium in this investigation. In this concept the soil pressure at a point is assumed to be directly proportional to the deflection at that point and the constant of proportionality is known as coefficient of subgrade reaction. In other words the soil is represented by infinitesimally small closely spaced independent elastic springs as shown in Fig. (1.2). The works of Biot (1937) and Vesic (1961) deal with the validity of this assumption for analysing the problem of beams on elastic foundation. It can be argued that the lateral deflections in the case of an axially loaded initially bent piles are small and thus it

may be assumed that at working loads, the coefficient of subgrade reaction remains constant with deflection at any depth. However, as pointed out by Chan and Hanna (1979) coefficient of subgrade reaction varies with depth which has been accounted for in this investigation.

In the past, as summarized in Chapter II, attempts have been made by Johnson (1962), Broms (1963), Kurma Rao (1975), Rao et al (1977) to develop simplified methods of arriving at allowable working or design loads assuming coefficient of subgrade reaction either constant with depth or linearly increasing with depth. No attempts have been made to study the behaviour of initially bent piles in non-homogeneous soil. This aspect is studied in this investigation.

In Chapter III initially bent bearing piles are analyzed for two layered soil system.

A layer coefficient is defined as the ratio of constant subgrade reactions of top and bottom layer. The initial shapes of the bent pile for three different end conditions viz. Fix-Fix, Fix-Pin, Pin-Pin are represented by polynomials satisfying both the natural and geometric boundary conditions. Closed form solutions are obtained for all the end conditions considered. The effect of length of pile, initial pile offset, layer coefficient, the top layer depth and the axial load on the additional deflections and additional bending moments are investigated. The load carrying capacities of the bent piles are expressed in

percentage load carrying capacities of the straight piles under similar conditions. Number of polynomials are considered to represent the initial shape of the bent pile for each case and unique relations are presented between initial radius of curvature and the percentage load carrying capacity for all the cases considered. Numerical data has been obtained comprehensively and results are presented in non-dimensional form.

In Chapter IV initially bent friction piles are analyzed by finite difference technique using Glesser's (1953) method as no closed form solution could be obtained for this case. The effect of soil friction on the additional deflections, additional bending moments and load carrying capacities are investigated. Also effect of linear and polynomial variation of coefficient of subgrade reaction on the capacities of initially bent friction pile in non-homogeneous soil is investigated. Down-drag effects are also investigated for few cases. Results are plotted in non-dimensional form.

Finally in Chapter V, conclusions are drawn from the investigations and suggestions for future research on this topic are given.

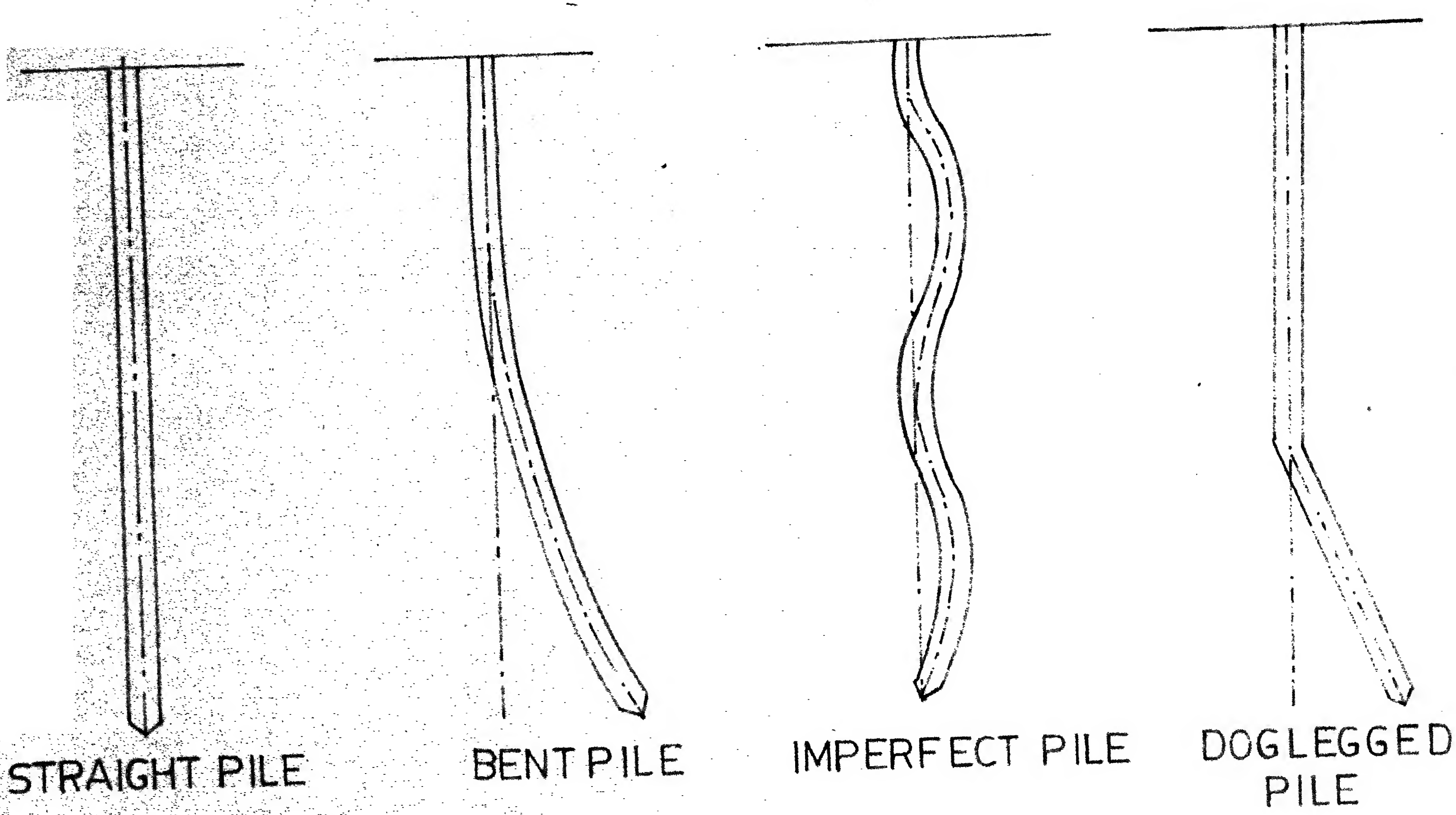


FIG.1.1 CLASSIFICATION OF PILES BASED ON THEIR INITIAL SHAPE (AFTER MADHAV,1976)

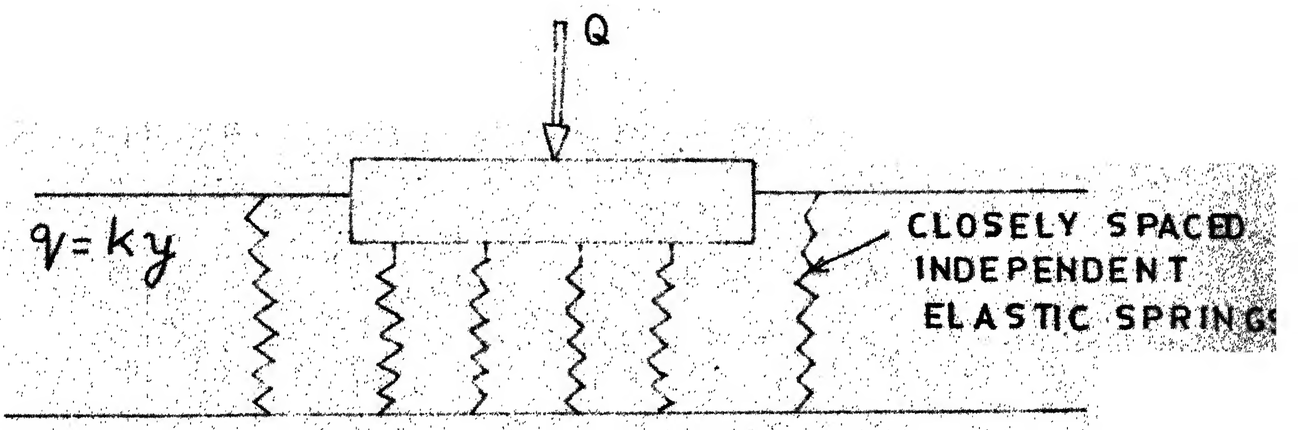


FIG.1·2 WINKLER MODEL

CHAPTER II

LITERATURE REVIEW

2.1 General:

Previous investigations on bent pile may be broadly classified in three groups: (1) Field and laboratory observation of pile bending, (2) attempts to assess the performance of bent piles and recommendations for design and (3) theoretical analyses to determine the pile capacity assuming certain initially bent shapes. By far, most of the reported investigation is on field observations starting about 25 years ago.

2.2 Field and Laboratory Observations:

Parsons and Wilsons (1954) reported a case where 800 end bearing piles of composite material were driven through a silty stratum to bed rock. Each pile was 40 m. in length the lower part being a 0.27 m. diameter steel tube and the upper part being 0.35 m. diameter corrugated shell. The piles were driven in preformed holes of limited depth and a heavy mandrel was used for driving the first 15 m. Slope indicator measurements showed the greater part of deviation from the vertical to be in the lower section of the piles with slopes exceeding 15°. At least 63 piles were bent to varying degrees,

the bases of some piles being upto about 2 m. from their intended position. An approximate method of assessing the safe loads was put forward, but this has limited general application in view of the assumptions made concerning the lateral soil pressure acting on the pile shaft.

According to Cummings (1956) piles are some times distorted into long pitch helical curves, some times into long sweeping bends and occassionally into sharp bends called dog-legs.

Johnson (1962) made field measurements on 27 m. long composite piles in sand and observed that the out of plan position of the pile base was of the order of approximately 10 percent of the pile length. Like Parsons and Wilsons (1954) he presented a method of safe load estimation that depended on a knowledge of the deflected shape of the pile and the load distribution along the pile shaft.

Perhaps the most serious deviations from plan position were observed in end bearing pre-cast piles driven in Gothenburg through a soft clay stratum to bed rock at 60 m. depth. The bases of these piles deviated by upto 11 m. from their intended position. The deflected shape of these piles which were made up of several segments was a gentle curve with the slope increasing with depth (National Swedish Council for Building Research, 1964).

Worth et al (1966) observed similar trends on much shorter piles driven through stiff clay and sand to a hard clay stratum. Also the distribution of load along the length of pile was measured.

Hanna (1968) reported the results of three tests on long H section steel piles driven through firm clays to bed rock at 44 m depth. A Wilson slope indicator was used to determine the plan position of these piles with depth. In contrast to the pile bending observations of others these piles had two bends and the field observations are reproduced in Fig. (2.1).

This diagram gives the measured components of pile deflections for two of these piles. In addition to these observations extensive load testing was carried out to arrive at a permissible load. Several points of practical significance emerged from this work. (1) From inspection of pile driving records it was not possible to decide if the pile bending had taken place. (2) Pile deviations were greater than normal pile to pile spacing in closely spaced groups, (3) When a bent pile is loaded overstressing of the pile material resulted. (4) The toe level of the bent piles is higher than that of a perfectly straight pile and end bearing piles may not be at bed rock horizon. (5) The initiation of the pile bending is due to the generation of asymmetrical stresses in the pile due to eccentric pile tip reactions and

eccentric driving inherent to all pile driving work. The reverse curve is due to primarily vertical weight component of inclined pile forcing the pile to bend.

There were instances in which two piles formed a conjunctive (U) so that one rose while that other was driven down and vice versa (Johnson, 1968).

Fellinius (1971) dealt with the bending of slender pre-cast piles which were driven in segments of 10 to 14 m. in length and spliced in the field by means of rigid steel couplings. An inclinometer for measuring the initial shape of bent pile was described and case studies are given for number of piles.

Fellinius (1972) dealt with the bending of slender pre-cast concrete piles with double curvature.

Kim et al (1973) conducted full scale lateral load tests on H piles of 13.3 m length. Twenty 10BP42 steel piles were driven to refusal. The piles were driven in three groups of six piles each with two isolated single piles (one vertical and one batter). Each pile was instrumented with 22 strain gauges and a 3.81 cm^2 steel tube to be used with a slope indicator. It was reported that the vertical piles deflected about the weak axis of the pile section during driving and the out of verticality was in the range of 0.15 m to 1.05 m. It was also observed that all piles were twisted during driving. The angle of twist ranged from 14° to 22° in eight piles, 26° to

40° in six other piles and 5° to 7° in the remaining piles. In general, the piles showed a tendency of twisting in the clock-wise direction.

Kim et al (1974) have described a new type of inclinometer, known as Digitilt-Minimose, which is a 22 mm diameter unit.Digitilt-Minimose is designed to be used in tubes that are smaller than the usual plastic or aluminium tubes, that accommodate the standing Digitilt Model. This new type is useful where multiple piles are joined by rigid caps.

Chan and Hanna (1979) reported the results of an experimental study at laboratory scale on the behaviour of bent (dog-legged) single piles when subjected to vertical loads. Varying degrees of bend were adopted for both friction and end bearing piles. The test piles which were embedded in cohesionless soils were instrumented to measure the distributions of axial load, bending moment and transverse shear along the pile shaft. Following conclusions are drawn from their experimental study.

1. The load settlement behaviour of bent friction piles is governed by the degree of bend.
2. A friction pile with large degree of bend has greater ultimate load than a straight or slightly bent pile.
3. Repeated loading appears to have considerable influence on the pile behaviour, particularly with respect to residual loads.

4. Large moments and shears are locked in the pile on unloading that can be directly related to the non-recoverable axial and lateral movements of the shift resulting from external loading.

5. When a pile is bent in one plane the axial loading of that pile causes two dimensional movements of the shaft. The tendency to undergo further bending diminishes with distance from the bend.

6. The coefficient of subgrade reaction for the initially bent piles increases with depth but decreases with horizontal pile movements.

A brief listing of above case histories is presented in Table 2.1.

From the above review it may be concluded that the pile bending or deviations during driving does occur in practice and it is not restricted to a particular soil type, pile system, or pile length. Some of the many inter-related factors effecting this process are discussed early in Chapter I.

2.3 Performance of Bent Piles and Design Recommendations:

In the second category of the reported work on bent piles, a few attempts have been made to provide acceptable rules for rejection of a pile that is out of straightness after driving. With hollow steel piles a rule of thumb has been used where by a tubular pile is rejected if part of the base cannot be seen when light is lowered inside it (Hanna,1970).

Norwegian Practice is to place a 50 mm diameter tube in the pile or welded along side of the pile in such a way that a small inclinometer may be lowered after driving (Bjerrum, 1957) and where the radius of curvature is less than 3000 m. the pile is usually rejected.

Swedish Practice recommends (Hellman, 1967) no reduction in the allowable load for concrete piles where the average radius of curvature of the pile axis is greater than 400 m over a length of 10 m, or 100 m. over a length of 2 m. For the steel pile the largest permissible deviation at a joint should not be more than 1 in 75 and the maximum allowable deviation is 1/1000 of the total pile length in the bottom 90 percent of the pile length. The maximum allowable deviation in top 10 percent of the pile length is 1/300 of the measuring distance.

2.4 Theoretical Analyses:

No significant theoretical work has been done on bent piles. Timoshenko (1936) was the first to obtain a solution for the case of a uniform pin-pin imperfect column in an elastic continuum which can be treated as a pile.

Glick (1948) presented an expression for buckling load taking the case of an imperfect pile bent in n half sine waves, assuming a differential equation for the elastic line of the pile.

Parsons and Wilsons (1954) presented solutions for finding safe loads on dog-legged piles.

The first systematic approach in the analysis of bent piles was presented by Johnson (1962) using Winklers hypothesis.

Broms (1963) presented a method for calculating the maximum lateral deflection, the maximum soil reaction and maximum bending moment for an axially loaded initially bent pile. He has in fact considered imperfect pile.

Marcus (as reported by Johnson, 1968) presented a solution for pinned-pinned initially bent pile . He was the first to show that the residual stresses due to bending are considerable. He used Winkler's hypothesis with constant subgrade modulus.

Francis et al (1965) have considered imperfections and loading eccentricities. They have shown that a pile with initial imperfections has the same buckling load as straight pile, however, the lateral deflections just before the critical load P_{cr} is reached, may be large.

Burgess (1975) has studied the directional instability caused by the "flutter" of a pile during driving. According to him the problem of a pile driven into a uniform soil is analogous to the aerodynamic problem of a cantilever with lateral elastic support, placed in a wind tunnel or flume,

aligned towards the flow. He predicted the critical depths for piles by studying the changes of the systems fundamental frequencies of oscillation when it is loaded by follower forces .

Madhav and Kurma Rao (1975) have presented a rational approach for the analysis smooth uniformly bent piles subjected to axial load for different end conditions using Winkler's hypothesis. They have also analysed imperfect piles for various end conditions. Methods for calculating the lateral load capacity of a bent pile have also been presented by them (1977) along with design charts. Rao et al (1975,1977) have presented solutions for cases of bent and imperfect piles. Kurma Rao (1975) analysed settlements of initially bent piles and the dog-legged piles.

Arumugam (1977) used elastic continuum model for the analysis of initially bent and imperfect piles. He emphasized the significance of initial shape of bent pile on the load carrying capacity by taking number of initial shapes formed by the first root and second root of the transcendental equation for pin-pin case. He also considered the shapes formed by the combination of the first root and second root. He applied reliability analysis to the initially bent and imperfect piles.

Omar (1980) presented a theoretical and experimental study on the directional stability of the driven piles. He

considered the flexural and torsional stresses and investigated the stability of the path followed by the pile. He pointed out that skin friction, horizontal subgrade modulus, the distribution of these parameters with depth and point resistance influence stability of the pile.

In summary, the previous investigations on bent piles can be classified into three categories. In the first category of work Parsons and Wilsons (1954), Johnson (1962), Hanna (1968), Fellinius (1971), Kim et al (1973), Chan and Hanna (1979) and others reported the field and laboratory observation of pile bending. In the second category of work different institutions and authorities established thumb rules for the rejection of already bent piles. In the third category of work capacity of the already bent piles were evaluated theoretically by Glick (1948), Parsons and Wilsons (1954), Johnson (1962), Broms (1963), Kurma Rao (1975), Rao et al (1977), Arumugam (1977) and others. Burgess (1975), Omar (1980) and others have worked on the directional stability of piles.

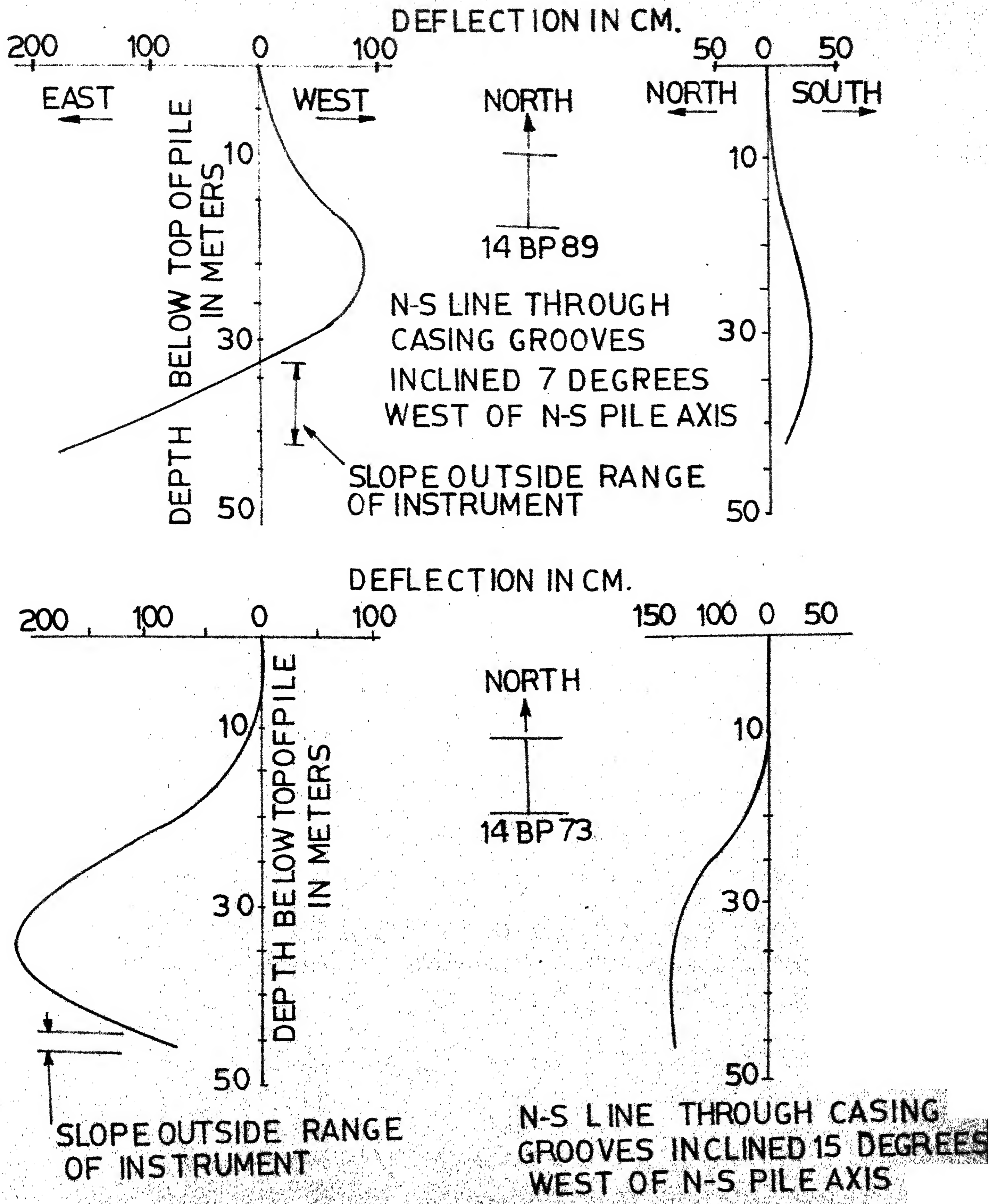
A review of the literature available on the initially bent piles suggests that no work has been done on the initially bent piles in non-homogeneous soils which is the situation often arises.

Table 2.1: Summary of reported pile bending measurements

Deviation of base, C(m)	Pile length, L(m)	C/L (%)	L/D[α]	Pile and soil details	Source
1.37	18.0	7.6	55	End bearing composite[+] pile, through silt to schist bedrock	Parsons and Wilson (1954)
3.04	27.4	11.1	75	Friction composite pile, in sand	Johnson (1962)
3.19	25.9	12.3	78	End bearing composite pile, through silt to dense sand	Mohr (1963)
11.0	60.0	18.3	218	End bearing precast concrete pile, through soft clay to bedrock	National Swedish Council for Building Research (1964)
1.85	24.6	7.5	69	Friction steel H-pile, through stiff clay and dense sand to hard clay	Worth et al.(1966)
2.16	42.6	5.1	120	End bearing steel H-pile, through soft to stiff clay, to shale bedrock	Hanna (1968)
2.65	40.0	6.6	133	End bearing precast concrete pile, through soft clay to mudstone bedrock	Fellenius (1972)
1.06	12.0	8.8	47	End bearing steel H-pile, through soft to firm clay, to limestone bedrock	Kim and Brungraber (1974)

α D = diameter or width of pile.

\+ Steel pipe filled with concrete after driving.



G.2·1 DEFLECTION COMPONENTS FOR 14 BP 89 AND 14 BP 73 PILES MEASURED APPROXIMATELY NORMAL TO AND PARALLEL TO WEAK AXIS OF EACH PILE (AFTER HANNA, 1968)

CHAPTER III

AXIAL LOAD CAPACITY OF INITIALLY BENT END BEARING PILES IN LAYERED SOIL

3.1 General:

The effect of two layered soil system on the flexural behaviour and the load carrying capacity of initially bent pile is investigated analytically in nondimensional terms. A modulus of subgrade reaction is used to define the soil stiffness. The stiffness of the bottom layer is assumed to vary in some constant proportion of top layer.

Theoretical solutions for the initially bent piles in soils having constant subgrade modulus along the pile length were given by Johnson (1962), Broms (1963) Madhav and Kurma Rao (1975). In practice seldom a soil stratum is encountered which has a constant soil profile. To take this into account the present investigation includes the analysis for soil profile which has varying modulus of subgrade reactor-with depth.

The analysis presented herein provides a qualitative study on the effects of soil layering on the initially bent end bearing piles. Two layered system is considered in this analysis, however, the same analysis can be extended to any number of layers.

3.2 <u>Capacity of Straight Steel Pipe Pile:</u>

A pile can fail either due to adjoining soil failure or due to structural failure. The capacity of pile due to soil failure depends on shear strength characteristics of the soil and various other factor which are discussed in many standard Soil Mechanics books and shall not be discussed in this thesis.

The problem of selecting allowable stress values for piling is a very complex one. In practice two relatively simple approaches are used. One is to assign fixed values of allowable stress to pile material and other is to consider pile as a column. Using the concept of pile as a column is a rational approach and most widely adopted by practising engineers. In this analysis this concept is used for calculating the allowable stresses in pile material. The design of a pile column is a function of six basic parameters.

1. The yield strength of the material.
2. The stiffness of material.
3. The equivalent unbraced length.
4. The amount of load P acting on the column.
5. The eccentricity of the load.
6. The cross-sectional area A of the column.

The load carrying capacity of straight steel pile is governed by the allowable stress in the column. IS-800 (1962)

has adopted the secant formula as the basis and the factor of safety adopted is 1.68. A similar but simple formula is given by American Institute of Steel Construction. According to AISC specifications 1962,

$$F_a = \frac{[1 - (\frac{KL}{r})^2/2C_c^{\ 2}]\ F_y}{F_s} \qquad \text{for } \frac{KL}{r} \leq C_c \qquad (3.1)$$

$$F_a = \frac{10.476\text{x}10^6}{(KL/r)^2} \qquad \text{for } \frac{KL}{r} \geq C_c \qquad (3.2)$$

where, F_a = axial stress permitted in the absence of bending moment

K = effective length factor

L = actual unbraced length

r = governing radius of gyration

C_c = Euler ratio = $[2\pi^2 E/F_y]^{1/2}$

E = Youngs' Modulus of the pile material

$$F_s = \text{factor of safety} = \frac{5}{3} + \frac{3(\frac{KL}{r})}{8C_c} - \frac{(\frac{KL}{r})^3}{8C_c^{\ 3}} \qquad (3.3)$$

F_y = specified minimum yield point for the type of steel being used.

Equation (3.1) applies where the ratio KL/r is less than the value of C_c. Equation (3.2) applies where the ratio KL/r is greater than C_c.

For the structural steel confirming to IS 226-1962, F_y = 2600 kg/cm^2, and $E = 2.047 \times 10^6$ kg/cm^2. In this

investigation the ratio of internal diameter to the external diameter of the pipe pile is tentatively taken as 0.9.

3.3 Capacity of Bent Pile:

As with straight pile the capacity of bent pile is checked from soil consideration as well as structural consideration of the pile material.

3.3.1 From Soil Considerations:

The capacity of bent piles in homogeneous soils depends on the strength characteristics of the soil material and a method for estimating its capacity is given by Broms (1963). In this method the maximum soil reactions should not exceed one third of the ultimate lateral resistance. The allowable soil reactions are given by,

Cohesive Soils - $q_{all} = 1.5\, q_u$ (3.4)

where q_u is unconfined compressive strength.

Cohesionless Soils- $q_{all} = \bar{p}_v \left(\frac{1 + \sin \phi d}{1 - \sin \phi d} \right)$ (3.5)

where $\bar{p}_v$ is the effective over burden pressure and ϕd is the angle of internal friction of the soil as measured from drained triaxial or direct shear test.

From soil consideration the allowable load on bent pile is given by,

$$P_{all} = \frac{q_{all}\, P_{cr}}{K_h\, u_{2_{max}} + q_{all}} \qquad (3.6)$$

where P_{cr} is the critical buckling load.

$u_{2_{max}}$ is maximum additional deflection along the length of the pile

K_h is coefficient of horizontal subgrade reaction.

q_{all} is allowable soil resistance.

Cummings (1938) obtained critical load of a pile hinged at both ends. He concluded that critical load exceeds the crushing strength of the pile unless the soil is excessively soft. He gave the following formula for the critical load.

$$P_{cr} = \frac{\pi^2 EI}{L^2} (2m^2 + 2m + 1) \tag{3.7}$$

where m is the number of half waves of the sinusoidal curve into which the pile tends to buckle.

L is the length of pile and

EI is the flexural rigidity of the pile.

Davisson (1963) presented charts for calculating the buckling loads for different end conditions.

Kurma Rao (1975) has given the following expressions for the critical buckling load of piles with fix-fix, fix-pin, pin-pin end conditions.

For fix-fix end condition,

$$P_{cr} = \frac{4\pi^2}{z_m^2} + \frac{z_m^2}{4\pi^2} \tag{3.8}$$

For fix-pin end conditions,

$$P_{cr} = \frac{\pi^2}{4z_m^2} + \frac{4z_m^2}{\pi^2} \qquad (3.9)$$

For pin-pin end conditions,

$$P_{cr} = \frac{(4.494)^2}{z_m^2} + \frac{z_m^2}{(4.494)^2} \qquad (3.10)$$

where,

z_m = L/T

L = length of pile

T = $[EI/KD]^{1/4}$ for cohesive soil.

T = $[EI/n_h]^{1/5}$ for cohesionless soil.

EI = flexural rigidity of the pile.

D = diameter of pile,

K = coefficient subgrade reaction, constant for cohesive soil.

n_h = coefficient of subgrade reaction for cohesionless soil.

3.3.2 From Structural Considerations:

AISC does not give specifications regarding the design of bent piles. However, it gives the design procedure for structural members subjected to combined compressive and bending stresses as given below:

When $\frac{f_a}{F_a} \leq 0.15$ $\qquad \frac{f_a}{F_a} + \frac{f_b}{F_b} \leq 1.0 \qquad (3.11)$

When $\frac{f_a}{F_a} > 0.15$ $$\frac{f_a}{F_a} + \frac{C_m f_b}{(1 - \frac{f_a}{F'_e}) F_b} \leq 1.0 \qquad (3.12)$$

where,

f_a = computed axial stress

F_a = axial stress permitted in the absence of bending moment

f_b = computed compressive bending stress

F_b = compressive bending stress permitted if bending moment alone existed

C_m = coefficient dependent on column curvature caused by applied moment

F'_e = $\frac{10.476 \times 10^6}{(KL_b/r^2)}$

K = effective length factor in the plane of bending

L_b = actual unbraced length in the plane of bending

r_b = radius of gyration about the axis of concurrent bending

It is assumed that the cross-sectional shape of the pile remains constant before and after bending. C_m varies from 0.85 to 1.0 and in this investigation C_m is tentatively taken as 1.0. To simplify the analysis it is further assumed that f_a/F_a is always greater than 0.15. This is justified as the axially loaded initially bent piles are being considered. Accordingly Eqn. (3.12) is reduced to:

$$\frac{S_1 + S_2}{F_b \left(1 - \frac{S_3 + S_2}{F'_e}\right)} + \frac{S_3}{F_a} \leq 1 \tag{3.13}$$

where,

S_1 is the flexural stress due to initial bending of the pile

S_2 is the flexural stress due to additional bending of the pile

S_3 is the axial stress in the pile

3.4 Statement of the Problem:

An idealized bent pile in a layered soil is illustrated in Fig. (3.1). Under the action of the axial load P there is a tendency for the pile to bend and deflect as shown by the dashed lines. The relative curvature, the magnitude of the displacements, and the number of nodes is a function of the pile length L, rigidity of the pile EI and the soil modulus of the foundation material K. u_1, q_1, M_1 and S_1 are the lateral displacement, the lateral pressure, the bending moment, and the shear force in the pile prior to the application of load P. The pile is assumed to be end bearing in which the entire applied load P is transmitted to the tip of the pile u_2, q_2, M_2 and S_2 are respectively, the additional lateral displacement, lateral pressure, bending moment and shear force on the pile due to the addition of load P.

The soil reaction q at any depth y is expressed as a linear function of the lateral deflection u at that point i.e.

$q = k\,u$, where k is coefficient of subgrade reaction and u is the displacement. This represents the assumption that the beam column in a soil medium may be treated analytically as a beam column restrained by a series of infinitely closely spaced independent elastic springs with stiffness k. Vesic (1961) presented several examples to show the validity of this assumption. Fig. (3.2) shows some of the variations of the soil modulus with depth for different types of soils and the one assumed in present investigation.

In analyzing a layered soil system the soil modulus of the bottom layer will be expressed in terms of the soil modulus of the top layer. Both relatively soft and relatively hard upper layers are considered. Fig. (3.3) shows the variation of soil modulus with depth that has been assumed in the analysis. The soil modulus of the top and bottom layers are K_t and K_b respectively. The ratio K_b/K_t is termed as layer coefficient R_o.

3.5 Assumptions and the Governing Differential Equation:

The following assumptions are made in the analysis.

1. The pile is of uniform cross-section with homogeneous material and is bent in a single plane with a smooth curvature and can be idealized as beam.
2. The moments, shears and displacements caused by the forces q acting on a curved beam are determined by applying

the same forces to a straight beam of the same length. This assumption has the same effect as determining the moments, and so forth, in an arch as though the arch were a beam. The computed moments and so forth will be greater than those that actually exist in the structure. The assumption therefore gives conservative results.

3. Failure of the pile material take place first since the initial stresses due to pile bending could be considerable.

4. The axial force in the pile along the length remains constant.

The first two assumptions are inherent in the derivation of the governing differential equation for the behaviour of axially loaded initially bent pile as given by Johnson (1968). The fourth assumption is valid only for this chapter.

Johnson (1968) and Rao et al (1975) have previously derived the differential equation to describe the behaviour of initially bent piles from equilibrium considerations for the cases of K equal to constant with depth and K varying linearly with depth respectively. Eqn. (3.14) is the governing differential equation where the flexural stiffness EI, and the axial load P are constant along the length of the pile for the case of constant soil modulus with depth.

$$EI \frac{d^4 u_2}{dy^4} + P \frac{d^2 u_2}{dy^2} + k D u_2 = -P \frac{d^2 u_1}{dy^2} \qquad (3.14)$$

Adopting conventional soil modulus $K = kD$, eqn. (3.14) can be written as,

$$EI \frac{d^4u_2}{dy^4} + P \frac{d^2u_2}{dy^2} + K u_2 = -P \frac{d^2u_1}{dy^2} \tag{3.15}$$

To make a linear differential equation K is assumed to be a function of y only. The resulting equation becomes a fourth order linear, non-homogeneous differential equation with constant coefficients which can be solved in the closed form.

3.6 Closed Form Solution:

The governing differential equation for the layered case can be written as,

$$EI \frac{d^4u_{2t}}{dy^4} + P \frac{d^2u_{2t}}{dy^2} + K_t u_{2t} = -P \frac{d^2u_1}{dy^2} \quad \text{for } 0 \leq y \leq L_1 \tag{3.16}$$

$$EI \frac{d^4u_{2b}}{dy^4} + P \frac{d^2u_{2b}}{dy^2} + K_b u_{2b} = -P \frac{d^2u_1}{dy^2} \quad \text{for } L_1 \leq y \leq L \tag{3.17}$$

where,

u_{2t} = Additional deflection of the pile in the upper layer due to axial load P

u_{2b} = Additional deflection of the pile in the bottom layer due to axial load P

K_t = Soil Modulus for top layer

K_b = Soil Modulus for bottom layer.

The eqns. (3.16) and (3.17) can be written in non-dimensional form by making use of non-dimensional parameters given by Matlock and Reese (1960).

A relative stiffness factor T is defined as

$$T = \left[\frac{EI}{K_t} \right]^{1/4}$$

Since the additional deflections are a function of pile material and its cross-section together with stiffness of supporting soil it is proposed to non-dimensionalize the pile length, initial deflection and additional deflection with respect to T.

Non dimensionalized depth coefficient $z = y/T$

Non dimensionalized length of pile $z_m = L/T$

Non dimensionalized thickness of top layer $= z_1 = L_1/T$

Non dimensionalized pile offset $= C/T$

Non dimensionalized initial deflections $w_1 = u_1/T$

Non dimensionalized additional deflections are

$$w_{2t} = u_{2t}/T, \quad w_{2b} = u_{2b}/T$$

Substituting above non dimensional parameters in the eqns.(3.16) and (3.17) they can be written in non-dimensional form as,

$$\frac{d^4 w_{2t}}{dz^4} + \alpha \frac{d^2 w_{2t}}{dz^2} + w_{2t} = -\alpha \frac{d^2 w_1}{dz^2} \quad \text{for } 0 \leq z \leq z_1 \tag{3.18}$$

$$\frac{d^4 w_{2b}}{dz^4} + \alpha \frac{d^2 w_{2b}}{dz^2} + R_o w_{2b} = -\alpha \frac{d^2 w_1}{dz^2} \quad \text{for } z_1 \leq z \leq z_m \tag{3.19}$$

where,

$$\alpha = \frac{PT^2}{EI} \quad \text{and} \quad R_o = \frac{K_b}{K_t}$$

Three pile boundary conditions such as fixed-fixed, fixed-pinned, pinned-pinned are analyzed. The initial shape, shear force and bending moment for each case are shown in Fig. (3.4).

3.6.1 Pile with Fixed-Fixed end Conditions:

In dimensionless form, the initial shape of initially bent pile with fixed-fixed boundary conditions may be represented by,

$$w_1 = \frac{C}{T}\left[1.25\frac{z^2}{z_m^2} + 5.0\frac{z^3}{z_m^3} - 8.75\frac{z^4}{z_m^4} + 3.5\frac{z^5}{z_m^5}\right] \quad (3.20)$$

where C/T is the non-dimensional offset of the pile tip from the vertical axis. The function w_1 needs to satisfy both geometric and natural boundary conditions of the problem. The geometric boundary conditions refer to the deflection and slope and natural boundary conditions refer to bending moment and shear force at the boundaries. Hoff (1956) has pointed out that it is generally preferable to select functions that satisfy both the geometric and natural boundary conditions, because as smaller number of terms are sufficient for an acceptable engineering approximation.

Equation (3.20) satisfies explicit geometric boundary conditions,

$$(w_1)_{z=0} = 0 \, , \quad (w_1)_{z=z_m} = \frac{C}{T} \, , \quad \left(\frac{dw_1}{dz} \right)_{z=0} = 0,$$

$$\left(\frac{dw_1}{dz} \right)_{z=z_m} = 0$$

as well as the implicit natural boundary conditions,

$$\left(\frac{d^2w_1}{dz^2} \right)_{z=0} \neq 0 \, , \quad \left(\frac{d^2w_1}{dz^2} \right)_{z = z_m} \neq 0$$

$$\left(\frac{d^3w_1}{dz^3} \right)_{z=0} \neq 0 \, , \quad \left(\frac{d^3w_1}{dz^3} \right)_{z=z_m} \neq 0$$

Therefore, Eqn. (3.20) can be used to represent the initial shape of the pile with fixed-fixed end conditions.

Let the particular solutions of Eqn. (3.18) and (3.19) be respectively,

$$w'_{2t} = a_o + a_1z_1 + a_2z^2 + a_3z^3 \tag{3.21}$$

$$w'_{2b} = b_o + b_1z + b_2z^2 + b_3z^3 \tag{3.22}$$

where w'_{2t} is the additional deflection of the pile in the top layer and w'_{2b} is the additional deflection of the pile in the bottom layer. Substituting w_{2t} and w_{2b} with their derivatives in Eqns. (3.18) and (3.19) and comparing the coefficients of equal powers of z on both sides the constants a_o, a_1, a_2, a_3, b_o, b_1, b_2 and b_3 can be obtained as,

$$a_o = \frac{-2.5 \, \alpha}{z_m^2} \frac{C}{T} - \frac{210 \, \alpha^2}{z_m^4} \, , \quad b_o = \frac{-2.5 \, \alpha}{R_o z_m^2} \frac{C}{T} - \frac{210 \, \alpha^2}{R_o^2 z_m^4} \frac{C}{T}$$

$$a_1 = \frac{-30\alpha}{z_m^3}\frac{C}{T} + \frac{420\alpha^2}{z_m^5}\frac{C}{T} \quad , \quad b_1 = \frac{-30\alpha}{R_o z_m^3}\frac{C}{T} + \frac{420\alpha^2}{R_o^2 z_m^5}\frac{C}{T}$$

$$a_2 = \frac{105\alpha}{z_m^4}\frac{C}{T} \quad , \quad b_2 = \frac{105\alpha}{R_o z_m^4}\frac{C}{T}$$

$$a_3 = \frac{-70\alpha}{z_m^5}\frac{C}{T} \quad , \quad b_3 = \frac{-70\alpha}{R_o z_m^5}\frac{C}{T}$$

The solution for homogeneous part of Eqns. (3.18) and (3.19) are,

$$w''_{2t} = C_1^+ \sin(m_1 z)\cosh(m_2 z) + C_2 \cos(m_1 z)\sinh(m_2 z) + D_1 \sin(m_1 z)\sinh(m_2 z) + D_2 \cos(m_1 z)\cosh(m_2 z)$$

$$\text{for } 0 \leq z \leq z_1 \qquad (3.23)$$

$$w''_{2b} = C'_1 \sin(m'_1 z)\cosh(m'_2 z) + C'_2 \cos(m'_1 z)\sinh(m'_2 z) + D'_1 \sin(m'_1 z)\sinh(m'_2 z) + D'_2 \cos(m'_1 z)\cosh(m'_2 z)$$

$$\text{for } z_1 \leq z \leq z_m \qquad (3.24)$$

where,

$$m_1^2 = 0.5 + \frac{\alpha}{4} \quad \text{and} \quad m_2^2 = 0.5 - \frac{\alpha}{4}$$

$$m_1^2 = \left(\frac{R_o}{4}\right)^{1/2} + \frac{\alpha}{4} \quad \text{and} \quad m_2^2 = \left(\frac{R_o}{4}\right)^{1/2} - \frac{\alpha}{4}$$

The total solution of the differential equations (3.18) and (3.19) is given by,

$$w_{2t} = w'_{2t} + w''_{2t}$$

$$w_{2b} = w'_{2b} + w''_{2b}$$

The constants C_1, C_2, D_1, D_2 and C_1', C_2', D_1', D_2' can be evaluated from the following 4 boundary conditions and four compatibility conditions.

At $z = 0$ $\quad w_{2t} = 0 \quad$ and $\quad dw_{2t}/dz = 0$

At $z = z_1$ $\quad w_{2t} = w_{2b}$

$$\frac{dw_{2t}}{dz} = \frac{dw_{2b}}{dz}, \qquad \frac{d^2w_{2t}}{dz^2} = \frac{d^2w_{2b}}{dz^2}$$

$$\frac{d^3w_{2t}}{dz^3} + \alpha \frac{dw_{2t}}{dz} = \frac{d^3w_{2b}}{dz^3} + \alpha \frac{dw_{2b}}{dz}$$

At $z = z_m$ $\quad w_{2b} = 0, \quad dw_{2b}/dz = 0$

In the case of bent piles, three types of stresses namely, the residual stress , the bending stress under load P and axial stresses are acting and their effect is cumulative. The various expression for stresses are obtained by making use of general elastic flexure equation of beams.

Residual stress

$$S_1 = \frac{E}{2}\frac{D}{T}\frac{C}{T}\left[\frac{2.5}{z_m^{\,2}} + 30\frac{z}{z_m^{\,3}} - 105\frac{z^2}{z_m^{\,4}} + 70\frac{z^3}{z_m^{\,5}}\right] \qquad (3.25)$$

Flexural stresses in the top layer $S_2 = \frac{E}{2}\frac{D}{T}\,[6a_3z + 2a_2$

$$-\sin(m_1z)\sinh(m_2z)\,\{D_1(m_2^{\,2}-m_1^{\,2}) - 2D_2m_1m_2\}$$

$$-\cos(m_1z)\cosh(m_2z)\,\{D_2(m_2^{\,2}-m_1^{\,2}) + 2D_1m_1m_2\}$$

$$-\sin(m_1 z)\cosh(m_2 z)\{C_1(m_2{}^2-m_1{}^2) - 2C_2\, m_1 m_2\}$$
$$-\cos(m_1 z)\sinh(m_2 z)\{C_2(m_2{}^2-m_1{}^2) + 2C_1\, m_1 m_2\}]$$
(3.26)

Flexural stresses in the bottom layer $S_2 = \frac{E}{2}\frac{D}{T}[6b_3 z + 2b_2$

$$-\sin(m_1' z)\sinh(m_2' z)\{D_1'(m_2'^2-m_1'^2) - 2D_2'\, m_1' m_2'\}$$
$$-\cos(m_1' z)\cosh(m_2' z)\{D_2'(m_2'^2-m_1'^2) + 2D_1'\, m_1' m_2'\}$$
$$-\sin(m_1' z)\cosh(m_2' z)\{C_1'(m_2'^2-m_1'^2) - 2C_2'\, m_1' m_2'\}$$
$$-\cos(m_1' z)\sinh(m_2' z)\{C_2'(m_2'^2-m_1'^2) + 2C_1'\, m_1' m_2'\}]$$
(3.27)

The expressions for S_1 and S_2 are obtained by making use of elastic flexure equation for beams.

Axial stress $$S_3 = \frac{4P}{\pi(D^2-d^2)} \qquad (3.28)$$

where D is the external diameter, and

d is the internal diameter.

3.6.2 Pile with Fixed-Pinned end Conditions:

In dimensionless form the initial shape for fixed-pinned end conditions may be represented by,

$$w_1 = \frac{C}{T}\left[0.5\frac{z^2}{z_m{}^2} + 75\frac{z^3}{z_m{}^3} - 11.75\frac{z^4}{z_m{}^4} + 4.75\frac{z^5}{z_m{}^5}\right] \qquad (3.29)$$

where $\frac{C}{T}$ is the dimensionless offset of the pile tip and z_m is the dimensionless length of pile.

Eqn. (3.29) satisfies the explicit geometric boundary conditions,

$$(w_1)_{z=0} = 0 , \qquad (w_1)_{z=z_m} = \frac{C}{T} , \qquad \left(\frac{dw_1}{dz} \right)_{z=0} = 0$$

as well as the implicit geometric boundary conditions $\left(\frac{dw_1}{dz}\right)_{z=z_m} \neq 0$

The explicit natural boundary condition $\left(\frac{d^2w_1}{dz^2} \right)_{z=z_m} = 0$ is satisfied.

The implicit natural boundary conditions,

$$\left(\frac{d^2w_1}{dz^2}\right)_{z=0} \neq 0 , \quad \left(\frac{d^3w_1}{dz^3} \right)_{z=0} \neq 0 , \quad \left(\frac{d^3w_1}{dz^3} \right)_{z=z_m} \neq 0$$

are also satisfied.

Therefore, Eqn. (3.29) can be used to represent the initial shape of the bent Fix-Pin pile.

Assuming the particular solution to be of the form,

$$w_{2t} = a_0 + a_1 z + a_2 z^2 + a_3 z^3 \quad \text{for } 0 < z \leq z_1 \qquad (3.21)$$

$$w_{2b} = b_0 + b_1 z + b_2 z^2 + b_3 z^3 \quad \text{for } z_1 \leq z \leq z_m \qquad (3.22)$$

Substituting Eqns. (3.21) and (3.22) together with (3.29) in the Eqns. (3.18) and (3.19) respectively and equating the equal powers of z on both sides the constants a_0, a_1, a_2, a_3, b_0, b_1, b_2, and b_3 can be obtained as,

$$a_0 = \frac{-\alpha}{z_m^2} \frac{C}{T} - \frac{282\ \alpha^2}{z_m^4} \frac{C}{T} , \quad b_0 = \frac{-\alpha}{R_o\ z_m^2} - \frac{282\ \alpha^2}{R_o^2\ z_m^4} \frac{C}{T}$$

$$a_1 = \frac{-45\alpha}{z_m^3}\frac{C}{T} + \frac{570\,\alpha^2}{z_m^5}\frac{C}{T}, \qquad b_1 = \frac{-45\,\alpha}{R_o z_m^3} + \frac{570\,\alpha^2}{R_o^2 z_m^5}\frac{C}{T}$$

$$a_2 = \frac{141\,\alpha}{z_m^4}\frac{C}{T}, \qquad b_2 = \frac{141\,\alpha}{R_o\, z_m^4}\frac{C}{T}$$

$$a_3 = \frac{-95\,\alpha}{z_m^5}\frac{C}{T} \qquad b_3 = \frac{-95\,\alpha}{R_o\, z_m^5}\frac{C}{T}$$

The solution for the homogeneous part of the differential Eqns. (3.18) and (3.19) is given by Eqns. (3.23) and (3.24).

The total solution of the differential Eqns. (3.18) and (3.19) is given by,

$$w_{2t} = w_{2t}' + w_{2t}''$$

$$w_{2b} = w_{2b}' + w_{2b}''$$

The constants C_1, C_2, D_1, D_2 and C_1', C_2', D_1', D_2' can be evaluated from the following four boundary condition and four compatibility conditions.

At $z = 0$ $\qquad w_{2t} = 0 \quad$ and $\quad \dfrac{dw_{2t}}{dz} = 0$

At $z = z_1$ $\qquad w_{2t} = w_{2b}, \quad \dfrac{dw_{2t}}{dz} = \dfrac{dw_{2b}}{dz}, \quad \dfrac{d^2w_{2t}}{dz^2} = \dfrac{d^2w_{2b}}{dz^2}$

$$\frac{d^3w_{2t}}{dz^3} + \alpha\frac{dw_{2t}}{dz} = \frac{d^3w_{2b}}{dz^3} + \alpha\frac{dw_{2b}}{dz}$$

At $z = z_m$ $\qquad w_{2b} = 0, \quad \dfrac{d^2w_{2b}}{dz^2} = 0$

Knowing the deflected shape under load P, the various expression for stresses can be expressed as follows.

Residual stress,

$$S_1 = \frac{E}{2}\frac{D}{T}\frac{C}{T}\left[\frac{1}{z_m^2} + 45\frac{z}{z_m^3} - 141\frac{z^2}{z_m^4} + 95\frac{z^3}{z_m^5}\right] \quad (3.30)$$

Flexural stress S_2 in the top and bottom layers are given by Eqn. (3.26) and (3.27) respectively.

Axial stress S_3 is given by Eqn. (3.28).

3.6.3 Pile with Pin-Pin End Conditions:

If a pile is just driven into a rigid stratum and its head is not fully encased it can be treated as a pin-pin pile.

The initial shape for pin-pin end conditions in dimensionless form may be represented as,

$$w_1 = \frac{C}{T}\left[1.5\frac{z}{z_m} - 7\frac{z^3}{z_m^3} + 11\frac{z^4}{z_m^4} - 4.5\frac{z^5}{z_m^5}\right] \quad (3.31)$$

Eqn. (3.31) satisfies the explicit geometric boundary conditions,

$$(w_1)_{z=0} = 0 \qquad \text{and} \qquad (w_1)_{z=z_m} = \frac{C}{T}$$

and also implicit geometric boundary conditions,

$$\left(\frac{dw_1}{dz}\right)_{z=0} \neq 0 \qquad \text{and} \qquad \left(\frac{dw_1}{dz}\right)_{z=z_m} \neq 0$$

The explicit natural boundary conditions,

$$\left(\frac{d^2 w_1}{dz^2} \right)_{z=0} = 0 \quad \text{and} \quad \left(\frac{d^2 w_1}{dz^2} \right)_{z=z_m} = 0$$

are also satisfied.

The implicit geometric boundary conditions,

$$\left(\frac{d^3 w_1}{dz^3} \right)_{z=0} \neq 0 \text{ and } \left(\frac{d^3 w_1}{dz^3} \right)_{z=z_m} \neq 0$$

are also satisfied.

Therefore Eqn. (3.31) can be used to represent the initial shape of the bent pin-pin pile.

Let the particular solution of the differential Eqns. (3.18) and (3.19) for pin-pin case be represented as,

$$w'_{2t} = a_0 + a_1 z + a_2 z^2 + a_3 z^3 \quad \text{for } 0 \leq z \leq z_1 \quad (3.21)$$

$$w'_{2b} = b_0 + b_1 z + b_2 z^2 + b_3 z^3 \quad \text{for } z_1 \leq z \leq z_m \quad (3.22)$$

Substituting Eqns.(3.21) and (3.22) together with the initial shape (3.31) in the governing differential equations (3.18) and (3.19) respectively and equating the equal powers of z on both sides the constants a_0, a_1, a_2, a_3 and b_0, b_1, b_2, b_3 can be obtained as,

$$a_0 = \frac{264\,\alpha^2}{z_m^4} \frac{C}{T}, \qquad b_0 = \frac{264\,\alpha^2}{R_o^2 z_m^4} \frac{C}{T}$$

$$a_1 = \frac{42\,\alpha}{z_m^3} \frac{C}{T} - \frac{540\,\alpha^2}{z_m^5} \frac{C}{T}, \quad b_1 = \frac{42\,\alpha}{R_o\, z_m^3} \frac{C}{T} - \frac{540\,\alpha^2}{R_o^2\, z_m^5} \frac{C}{T}$$

$$a_2 = -\frac{132\,\alpha}{z_m^4}\frac{C}{T}, \qquad b_2 = -\frac{132\,\alpha}{R_o\,z_m^4}\frac{C}{T}$$

$$a_3 = \frac{90\,\alpha}{z_m^5}\frac{C}{T}, \qquad b_3 = \frac{90\,\alpha}{R_o\,z_m^5}\frac{C}{T}$$

The solutions for the homogeneous part of the differential Eqns. (3.18) and (3.19) are given by Eqns. (3.23) and (3.24).

The total solution of the differential Eqns. (3.18) and (3.19) are given by,

$$w_{2t} = w'_{2t} + w''_{2t}$$

$$w_{2b} = w'_{2b} + w''_{2b}$$

The constants C_1, C_2, D_1, D_2 and C'_1, C'_2, D'_1, D'_2 can be evaluated from the following four boundary conditions and four compatibility conditions.

At $z = 0$ $\quad w_{2t} = 0$ and $\dfrac{d^2 w_{2t}}{dz^2} = 0$

At $z = z_1$ $\quad w_{2t} = w_{2b}$, $\dfrac{dw_{2t}}{dz} = \dfrac{dw_{2b}}{dz}$, $\dfrac{d^2 w_{2t}}{dz^2} = \dfrac{d^2 w_{2b}}{dz^2}$

$$\frac{d^3 w_{2t}}{dz^3} + \alpha\frac{dw_{2t}}{dz} = \frac{d^3 w_{2b}}{dz^3} + \alpha\frac{dw_{2b}}{dz}$$

At $z = z_m$ $\quad w_{2b} = 0$, $\dfrac{d^2 w_{2b}}{dz^2} = 0$

Knowing the deflected shape under load P, the various expressions for stresses can be expressed as follows:

Residual stress

$$S_1 = \frac{E}{2}\frac{D}{T}\frac{C}{T}\left[-42\frac{z}{z_m^{\;3}} + 132\frac{z^2}{z_m^{\;4}} - 90\frac{z^3}{z_m^{\;5}}\right] \qquad (3.32)$$

Flexural stresses S_2 in the top and bottom layers are given by Eqns. (3.26) and (3.27) respectively.

Axial stress S_3 is given by Eqn. (3.28).

3.7 Other Possible Shapes:

A bent pile need not necessarily assume the shapes as given above. In fact the pile bending and the types of stresses developed due to bending is a complex phenomena and is difficult to solve rigorously. Even for the most idealized case there can be number of shapes satisfying all the natural and geometric boundary conditions as suggested by Hoff (1956). Different initial shapes are assumed for the same length and initial pile off-set to study the effect of shapes on the load carrying capacity of initially bent pile.

The shape of an initially bent pile for any end conditions can be represented by the following generalized polynomial.

$$w_1 = \frac{C}{T}\left[a_0 + a_1\frac{z}{z_m} + a_2\frac{z^2}{z_m^{\;2}} + a_3\frac{z^3}{z_m^{\;3}} + a_4\frac{z^4}{z_m^{\;4}} + a_5\frac{z^5}{z_m^{\;5}}\right] \qquad (3.33)$$

In total six boundary conditions (two geometric boundary conditions and the natural boundary conditions at each end) enable to determine the six constants in the Eqn. (3.33).

3.7.1 Fix-Fix End Conditions:

The geometric boundary conditions to be satisfied for this case are:

$$\text{at } z = 0 \qquad w_1 = 0 \qquad \text{and} \qquad \frac{dw_1}{dz} = 0$$

$$\text{at } z = z_m \qquad w_1 = \frac{C}{T} \qquad \text{and} \qquad \frac{dw_1}{dz} = 0$$

The natural boundary conditions to be satisfied are:

$$\text{at } z = 0 \qquad \frac{d^2w_1}{dz^2} \neq 0 \qquad \text{and} \qquad \frac{d^3w_1}{dz^3} \neq 0$$

$$\text{at } z = z_m \qquad \frac{d^2w_1}{dz^2} \neq 0 \qquad \text{and} \qquad \frac{d^3w_1}{dz^3} \neq 0$$

Assuming arbitrarily the shear force at top and bottom of the bent pile as $6F_1$ and $6F_2$, (for mathematical convenience) and substituting the geometric boundary conditions in Eqn.(3.33), six simultaneous equations can be formed which can be solved to obtain six constants as given by,

$$a_0 = 0\,; \qquad a_1 = 0\,; \qquad a_2 = \frac{10-1.5F_1 + 0.5F_2}{4}$$

$$a_3 = F_1; \qquad a_4 = \frac{-10-3.5F_1-1.5F_2}{4}\,, \qquad a_5 = \frac{F_1+F_2+4}{4}$$

For each set of F_1 and F_2 values such that $20-3F_1+F_2 \neq 0$ and $20+F_1-3F_2 \neq 0$ an initial shape satisfying the natural and geometric boundary conditions for fix-fix end conditions can be formed.

3.7.2 Fix-Pin End Condition:

The geometric boundary conditions to be satisfied are:

$$\text{at } z = 0 \qquad w_1 = 0 \qquad \text{and} \qquad \frac{dw_1}{dz} = 0$$

$$\text{at } z = z_m \qquad w_1 = \frac{C}{T} \qquad \text{and} \qquad \frac{dw_1}{dz} \neq 0$$

The natural boundary conditions to be satisfied are:

$$\text{at } z = 0 \qquad \frac{d^2w_1}{dz^2} \neq 0 \qquad \text{and} \qquad \frac{d^3w_1}{dz^3} \neq 0$$

$$\text{at } z = z_m \qquad \frac{d^2w_1}{dz^2} = 0 \qquad \text{and} \qquad \frac{d^3w_1}{dz^3} \neq 0$$

Assuming the shear force at depth K_z equal to zero and the slope at $z = z_m$ equal to θ and substituting these conditions together with geometric boundary conditions in Eqn. (3.33). Six simultaneous equations can be formed which can be solved to obtain six constants as given by,

$$a_0 = 0\,; \qquad a_1 = 0; \qquad a_4 = \frac{(-4\theta+6)\ (30K_z^2-15)-6\theta}{60K_z^2 - 72K_z + 18}$$

$$a_5 = \frac{2\theta - (12K_z - 8)\ (-4\theta + 6)}{60K_z^2 - 72K_z + 18}$$

$$a_3 = (\theta-2) - 2a_4 - 3a_5\,; \qquad a_2 = 1 - a_3 - a_4 - a_5$$

For each set of K_z and θ values such that $a_2 \neq 0$, $a_3 \neq 0$, $a_3 + 4a_4 + 10a_5 \neq 0$ an initial shape satisfying the natural and geometric boundary conditions can be formed.

3.7.3 Pin-Pin End Conditions:

The geometric boundary conditions to be satisfied are:

$$\text{at } z = 0 \qquad w_1 = 0 \qquad \text{and} \quad \frac{dw_1}{dz} \neq 0$$

$$\text{at } z = z_m \qquad w_1 = \frac{C}{T} \qquad \text{and} \quad \frac{dw_1}{dz} \neq 0$$

The natural boundary conditions to be satisfied are:

$$\text{at } z = 0 \qquad \frac{d^2w_1}{dz^2} = 0 \qquad \text{and} \quad \frac{d^3w_1}{dz^3} \neq 0$$

$$\text{at } z = z_m \qquad \frac{d^2w_1}{dz^2} = 0 \qquad \text{and} \quad \frac{d^3w_1}{dz^3} \neq 0$$

Assuming the slopes at the top end and bottom end of the piles θ_1 and θ_2, respectively and substituting the geometric boundary conditions together with the boundary conditions of zero bending moment at the ends in Eqn. (3.33) results in the formation of six simultaneous equations which can be solved to obtain the six unknowns as given below.

$$a_0 = 0; \qquad a_1 = \theta\,; \qquad a_2 = 0\,; \qquad a_5 = 6-3\theta_1 - 3\theta_2\,;$$

$$a_4 = 8\theta_1 + 7\theta_2 - 15; \qquad a_3 = -6\theta_1 - 4\theta_2 + 10$$

For each set of θ_1 and θ_2 values such that $10-6\theta_1-4\theta_2 \neq 0$ and $10-4\theta_1-6\theta_2 \neq 0$ an initial shape satisfying the natural and geometric boundary condition can be formed.

3.8 Initial Curvature and Capacity of Bent Piles:

Hanna (1968) highlighted the effect of initial radius of curvature on the initial bending stresses in the bent pile. Kurma Rao (1975) has shown the effect of R/D ratio on the percentage load carrying capacity of bent pile using Winkler model where R is the initial radius of curvature and D is the diameter of the pile. Arumugam (1977) found using the elastic continuum model, the relation between R/D ratio and the percentage load carrying capacity of bent piles for pin -pin end conditions for different configurations to be unique.

Different configurations are formed by the combination of the first two roots of the transcendental equation. In this investigation, the effect of R/T ratio on the percentage load carrying capacity of bent steel pipe piles having different configurations is studied for fix-fix, fix-pin and pin-pin end conditions using Winkler model.

From calculus the expression for radius of curvature can be written in non-dimensional form as,

$$\frac{R}{T} = \frac{\left(1 + \left(\frac{dw_1}{dz}\right)^2\right)^{3/2}}{\frac{d^2w_1}{dz^2}}$$

The value of minimum radius of curvature along the length of the pile is chosen and the corresponding value of the capacity of bent steel pipe pile is evaluated as explained previously.

3.9 Results and Discussion:

To study the effect of axial load on additional deflections (w_2) and bending moments (M_2) the parameters $z_m = 5.0$, $R_o = 1.0$, $\frac{C}{T} = 0.01$ and $\frac{T}{r} = 22.0$ are kept constant and the load on the pile is varied. α_c is the load carrying capacity of straight pile under similar conditions. Figs. 3.5a, 3.5b and 3.5c show the variation of additional deflections (w_2) for fix-fix, fix-pin, pin-pin end conditions respectively. Figs. 3.6a, 3.6b and 3.6c show the variation of additional bending moments (M_2) for fix-fix, fix-pin and pin-pin end conditions respectively. It is observed that for all the end conditions the deflections and bending moments are directly proportional to the load on the pile. For fix-fix end conditions the maximum additional deflection (w_2) occurs at a depth of 1.25 and 3.75 simultaneously where as the maximum additional bending moments occur at the ends and also at depths of 1.25 and 3.75. For fix-pin end conditions maximum addition deflection (w_2) occurs at a depth of 3.75 whereas maximum additional bending moment (M_2) occurs at the top end and at a depth of 3.75 simultaneously. For pin-pin end conditions both maximum additional deflection and bending moment occur at a depth of 3.75.

To study the effect of initial off-set on the additional deflections (w_2) and bending moments (M_2) the parameters $z_m = 5.0$ $R_o = 1.0$, $\alpha = 0.75\alpha_c$ and $T/r = 22.0$ are kept constant and C/T is varied. Fig. 3.7a, 3.7b, and 3.7c show the variation of

additional deflections (w_2) for fix-fix, fix-pin and pin-pin end conditions respectively. The corresponding figures in Figs. 3.8a, 3.8b and 3.8c show the variation of additional bending moments (M_2) for the respective end conditions. As shown in all the figures the additional deflections (w_2) and additional bending moments (M_2) are directly proportional to initial off-set of the pile. The positions of maximum deflections and maximum bending moments are same as for previous case.

To study the effect of layering on the variation of additional deflections (w_2) and bending moments (M_2) the parameters $z_m = 5.0$, $z_1 = 2.5$, $C/T = 0.01$, and $T/r = 22.0$ are kept constant and R_o is varied. Figs. 3.9a, 3.9b and 3.9c show the variation of additional deflections (w_2) with the change in strength of bottom layer (R_o) for fix-fix, fix-pin and pin-pin end conditions respectively. Figs. 3.10a, 3.10b, 3.10c show the variation of additional bending moments (M_2) with R_o for fix-fix, fix-pin and pin-pin end conditions respectively. As expected when the bottom layer is weaker than the top layer ($R_o = 0.25$) the bending moments (M_2) and additional deflections (w_2) increases. The increase in additional deflections (w_2) and bending moments (M_2) is higher in the bottom weaker layer compared to top stronger layer. In contrary when the bottom layer is stronger than the top layer X($R_o = 4.0$) the deflections (w_2) and bending moments (M_2) decrease all along the length of

pile and the maximum reduction is found to be in stronger layer.

The capacity of few bent piles from the soil failure considerations are calculated using the formula given by Broms (1963) and buckling loads given by Kurma Rao (1975). A typical example is shown below.

For a steel pile of length 11.22 m, external diameter 30 cm, internal diameter 27 cm with an initial off-set of 2.24 cm embedded in a homogeneous cohesive soil with an undrained shear strength of 0.34 kg/cm^2 the nondimensional parameters can be obtained as follows.

Moment of inertia I $= \frac{\pi}{64}(30^4 - 27^4) = 13673.73$ cm^4

Area of cross-section A $= \frac{\pi}{4}(30^2 - 27^2) = 134.3$ cm^2

Radius of gyration r $= (I/A)^{1/2} \doteq 10.09$ cm

Terzaghi (1955) has given the relation between coefficient of horizontal subgrade reaction (t/ft^3) and undrained shear strength (t/ft^2) as K = 33 Cu.

This can be written as,

$$K = 1.1\ Cu$$

where, K is coefficient of horizontal subgrade reaction in kg/cm^3.

Cu is undrained shear strength in kg/cm^2.

$\therefore$ Relative stiffness factor

$$T = \sqrt[4]{\frac{EI}{KD}} = \sqrt[4]{\frac{2.047\times10^{6}\times13673.73}{(1.1\times0.34)\ 30}} = 223.5 \text{ cm}$$

The non-dimensional parameters can be obtained as,

$$z_m = 5.02, \qquad \frac{C}{T} = 0.01, \qquad \frac{T}{r} = 22.15$$

$$\simeq 5.00 \qquad\qquad\qquad \simeq 22.00$$

The details of calculations for obtaining the capacity of bent pile from soil failure considerations are shown in Table 3.1. All the parameters shown are dimensionless.

It can be seen from the above example that the load carrying capacity of bent pile from soil failure criteria is much higher than the capacity calculated from structural failure criteria. Hence in this investigation only structural capacities of the bent piles have been obtained.

Fig. 3.11a, 3.11b and 3.11c show the variation of load carrying capacity with z_m for fix-fix, fix-pin and pin-pin end conditions respectively. The parameters $R_o = 1.0$ and $T/r = 22.0$ are kept constant. It can be seen from the figure that for $C/T = 0.01$ there is no significant change in the pile capacity after $z_m = 5.0$. However, for higher off-set values longer piles provide higher load capacities.

Fig. 3.12a, 3.12b and 3.12c show the relation between load carrying capacity of bent pile with C/T for fix-fix, fix-pin and pin-pin end conditions respectively. The parameters

$R_o = 1.0$ and $T/r = 22.0$ are kept constant. It is observed in all the figures that the load carrying capacity reduces very rapidly for short piles with increasing off-set.

Fig. 3.13a, 3.13b and 3.13c show the relation between C/L, initial off-set expressed in terms of pile length, against capacity of bent pile for fix-fix, fix-pin, pin-pin end conditions respectively. At smaller pile offsets the behaviour is not well defined but at higher offsets the capacities of longer piles are larger than shorter piles.

Fig. 3.14a, 3.14b and 3.14c show the effect of slenderness parameter T/r on the percentage load carrying capacities of bent piles with fix-fix, fix-pin and pin-pin end conditions respectively. The parameters $z_m = 5.0$, $R_o = 1.0$ are kept constant. At larger offsets percentage load carrying capacity increases as the slenderness parameter increases. This increase is attributed to lower load carrying capacities of straight end bearing piles. However, for smaller offsets, there appears to be a slenderness parameter at which capacity will be maximum.

Fig. 3.15a, 3.15b, 3.15c show the effect of R_o on the load carrying capacity of bent pile with fix-fix, fix-pin, and pin-pin end conditions respectively. The parameters $z_m = 5.0$, $z_1 = 2.50$, $C/T = 0.01$ and $T/r = 22.0$ are kept constant. As seen in these figures for all end conditions the load carrying capacity increases with the increase of R_o due to increase in the strength of the bottom layer. However the

magnitude of increase is insiginficant. This may be due to high Young's Modulus of steel piles. If the initial pile offset is high the initial bending stresses would be high and hence allowable load and consequently the additional deflections would be small. If the initial offset is small the axially load capacity of bent pile would certainly be high but the eccentricity will be smaller and thus additional deflections and bending stresses would be small. Thus in both cases the small additional deflections would make the soil strength an insensitive parameter to the load carrying capacity of bent pile. For this reason we find that the effect of the strength of soil is insignificant on the load carrying capacity of bent steel piles.

Figs. 3.15d, 3.15e, 3.15f show the effect of the depth of top layer on the load carrying capacity of bent pile for fix-fix, fix-pin and pin-pin end conditions respectively. The parameters $z_m = 5.0$, $C/T = 0.01$ and $T/r = 22.0$ are kept constant. The parameter R_o is given values of 0.25 and 4.00 and z_1 is varied between 1.25 and 3.75. It can be seen from these figures that for all end conditions when $R_o = 0.25$ (i.e. when the bottom layer is weaker), increase in the thickness of top layer increases the load carrying capacity where as when $R_o = 4.0$ increase in the thickness of top layer decreases the load carrying capacity. However, the change in load carrying capacity with change in thickness of top layer is insignificant.

The effect of initial offset on the additional deflections of bent piles at the allowable load on the bent pile is shown in Fig. 3.16a, 3.16b, 3.16c for fix-fix, fix-pin, and pin-pin end conditions respectively. In this figure the additional deflections are expressed as a percentage of initial deflections for various values of z_m. It is seen that at larger offsets the percentage change in additional deflection is smaller than at smaller off-sets. For example, in fix-pin case for $z_m = 5.0$ a change in the value of C/T from 0.02 to 0.04 (i.e. 50 percent increase) reduced the percentage additional deflections from 1.0 percent to 0.4 percent (i.e. 60 percent reduction) in additional deflections). This clearly shows that the effect of initial bending stresses becomes predominant as C/T increases. For pin-pin end conditions the maximum additional deflections are negative. However, the increase in the magnitude additional deflections reduces at higher offset values. Fig. 3.17a, 3.17b, 3.17c show the additional bending moments as a percentage of initial bending moments for the same pile with fix-fix, fix-pin and pin-pin end conditions respectively.

It is recognized that the load carrying capacity of bent pile depends on the initial shape of the pile. To study this aspect number of shapes have been considered and analyses were conducted keeping initial offset constant, for fix-fix, fix-pin, pin-pin end conditions. For all shapes and end

conditions considered the parameters $z_m = 5.0$, $R_o = 1.0$, $T/r = 22.0$ are kept constant and C/T is varied between 0.01 to 0.05. Three typical shapes, shape I, shape II, shape III for each end condition are shown in Fig. (3.18). The load carrying capacities for shape I, shape II and shape III are evaluated with different initial offset values for all the end conditions and are shown in Fig. (3.19).

It is interesting to note that for all shapes and different offset values, a unique relationship is found between R/T and load carrying capacity for each end condition (Fig.3.20). For all end conditions the load carrying capacity increases as radius of curvature R/T increases. At a particular radius of curvature the capacity is maximum for fix-fix end conditions and minimum for pin-pin end conditions. Similar curves between R/D (Ratio of radius of curvature to diameter of the pile) and load carrying capacity were presented by Kurma Rao (1975), Arumugam (1977) which is independent of pile end conditions. In their analysis it is assumed that the allowable stress in the pile material remains constant irrespective of end conditions and length of the pile. However, in the present investigation the steel pipe pile design is based on the column concept in which allowable stresses changes with pile length and end conditions. For this reason different relations are obtained for different end conditions.

Table 3.1: Capacity of Bent Pile from Soil Failure Consideration.

	Fix-Fix	Fix-Pin	Pin-Pin
1. Critical buckling load given by Kurma Rao	2.212	10.15	2.33
2. Load carrying capacity of straight piles α_c	0.2768	0.2450	0.1971
3. % Load carrying capacity of bent pile from structural failure criteria	71.75%	64.98%	60.52%
4. Load carrying capacity of bent piles	0.1986	0.1592	0.1193
5. Maximum additional deflection corresponding to the load at structural failure	0.5609×10^{-4}	0.9402×10^{-4}	0.8572×10^{-4}
6. Allowable load from soil failure criteria	2.212	10.15	2.33

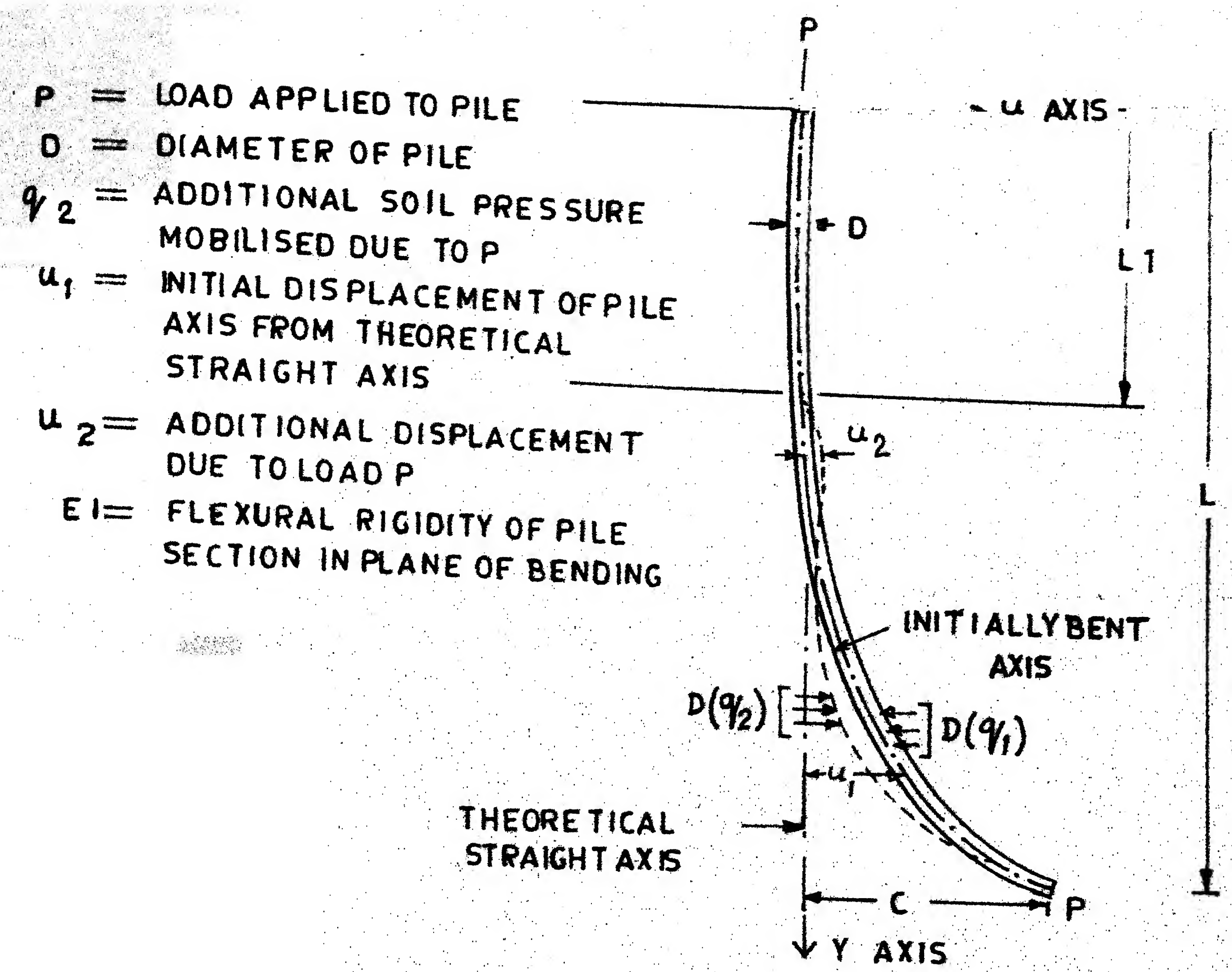
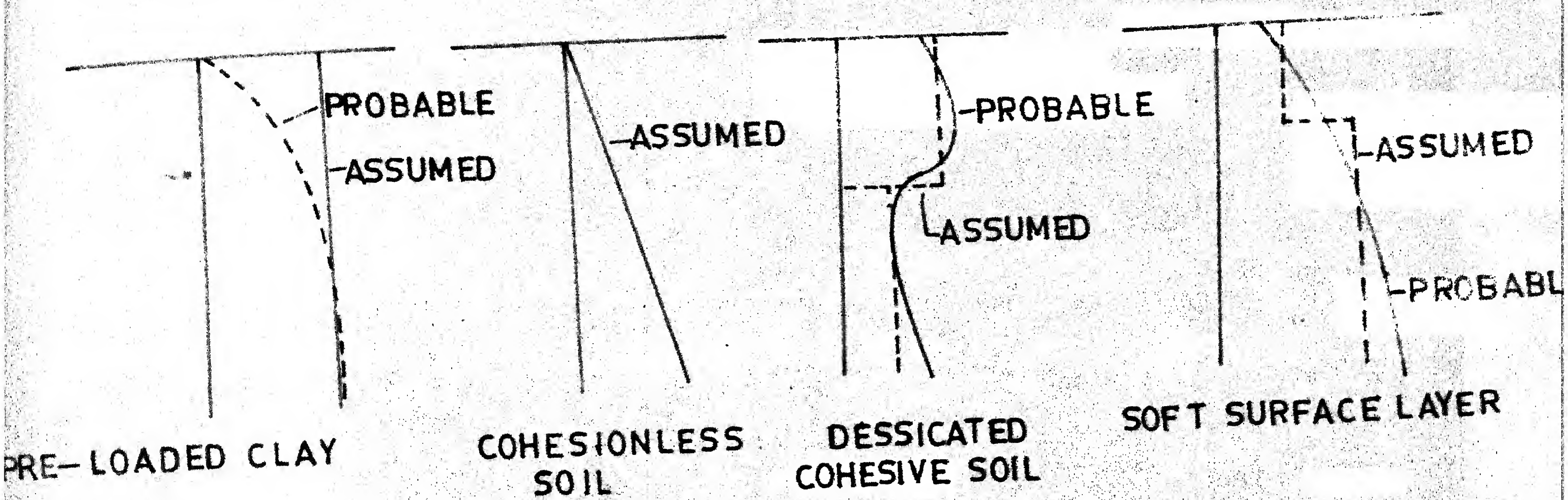


FIG.3·1 TERMINOLOGY



IG.3·2 VARIATION OF SOIL MODULUS WITH DEPTH
(AFTER DAVISSON AND GILL, 196

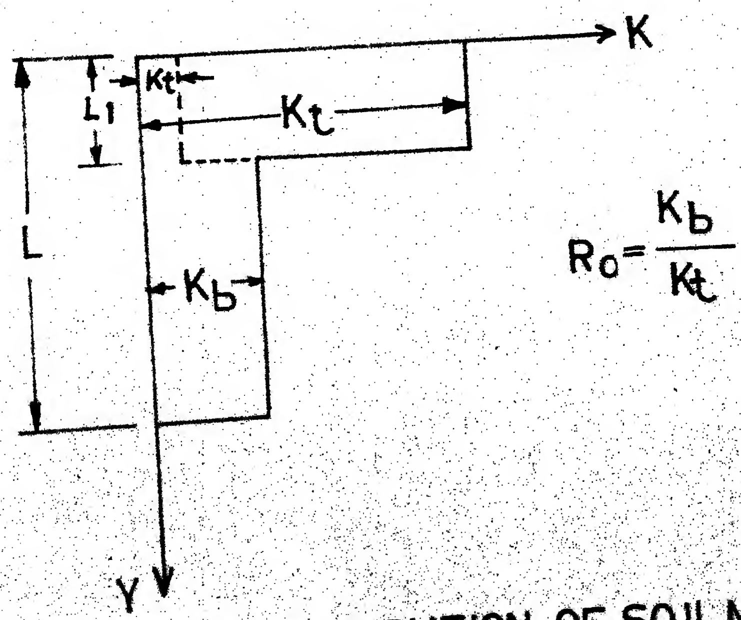
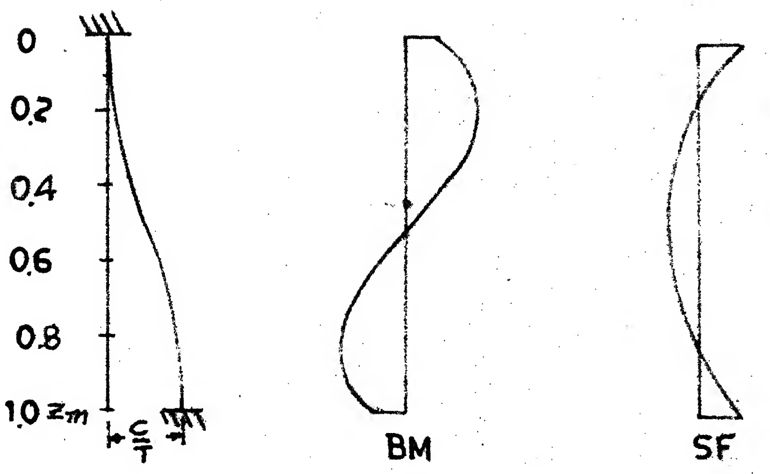


FIG. 3·3 IDEALISED VARIATION OF SOIL MODULUS

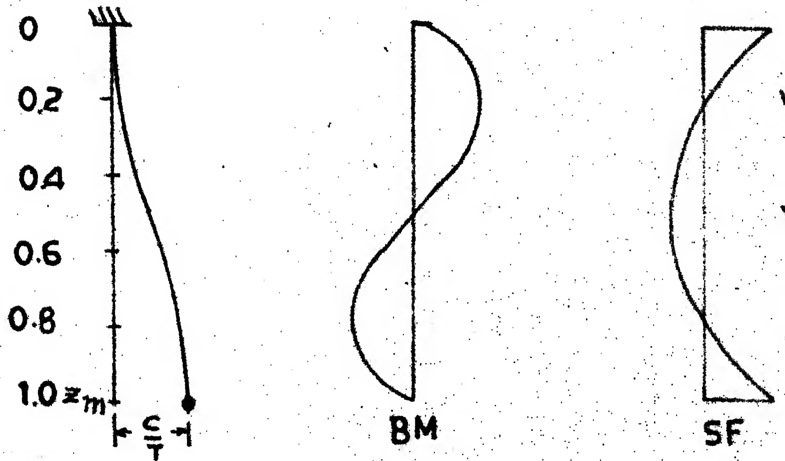


$$W_1 = \frac{C}{T}\left[1.25\frac{z^2}{z_m^2} + 5.0\frac{z^3}{z_m^3} - 8.75\frac{z^4}{z_m^4} + 3.5\frac{z^5}{z_m^5}\right]$$

$$W_1'' = \frac{C}{T}\left[2.5\frac{1}{z_m^2} + 30\frac{z}{z_m^3} - 105\frac{z^2}{z_m^4} + 70\frac{z^3}{z_m^5}\right]$$

$$W_1'' = \frac{C}{T}\left[\frac{30}{z_m^3} - 210\frac{z}{z_m^4} + 210\frac{z^2}{z_m^5}\right]$$

FIX-FIX

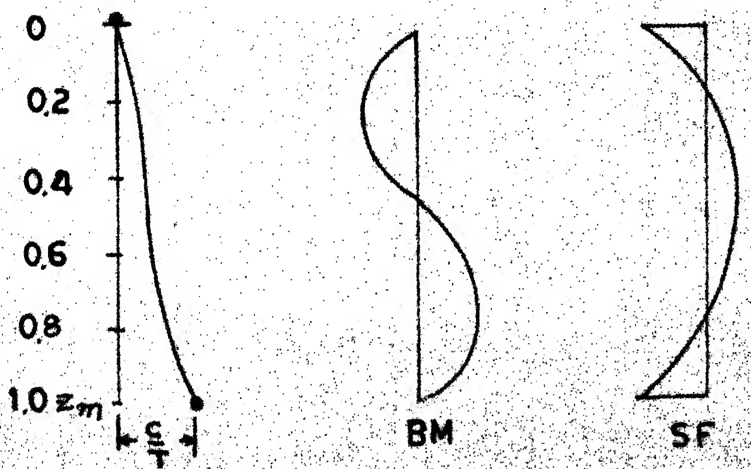


$$\dot{W}_1 = \frac{C}{T}\left[0.5\frac{z^2}{z_m^2} + 7.5\frac{z^3}{z_m^3} - 11.75\frac{z^4}{z_m^4} + 4.75\frac{z^5}{z_m^5}\right]$$

$$W_1'' = \frac{C}{T}\left[\frac{1}{z_m^2} + 45\frac{z}{z_m^3} - 141\frac{z^2}{z_m^4} + 95\frac{z^3}{z_m^5}\right]$$

$$W_1'' = \frac{C}{T}\left[\frac{45}{z_m^3} - 282\frac{z}{z_m^4} + 285\frac{z^2}{z_m^5}\right]$$

FIX-PIN



$$W_1 = \frac{C}{T}\left[1.5\frac{z}{z_m} - 7\frac{z^3}{z_m^3} + 11\frac{z^4}{z_m^4} - 4.5\frac{z^5}{z_m^5}\right]$$

$$W_1'' = \frac{C}{T}\left[-42\frac{z}{z_m^3} + 132\frac{z^2}{z_m^4} - 90\frac{z^3}{z_m^5}\right]$$

$$W_1''' = \frac{C}{T}\left[-\frac{42}{z_m^3} + 264\frac{z}{z_m^4} - 270\frac{z^2}{z_m^5}\right]$$

PIN-PIN

FIG. 3.4 PILE BUNDARY CONDITIONS CONSIDERED RESPECTIVE INITIAL SHAPE, BM AND SF DIAGRAMS

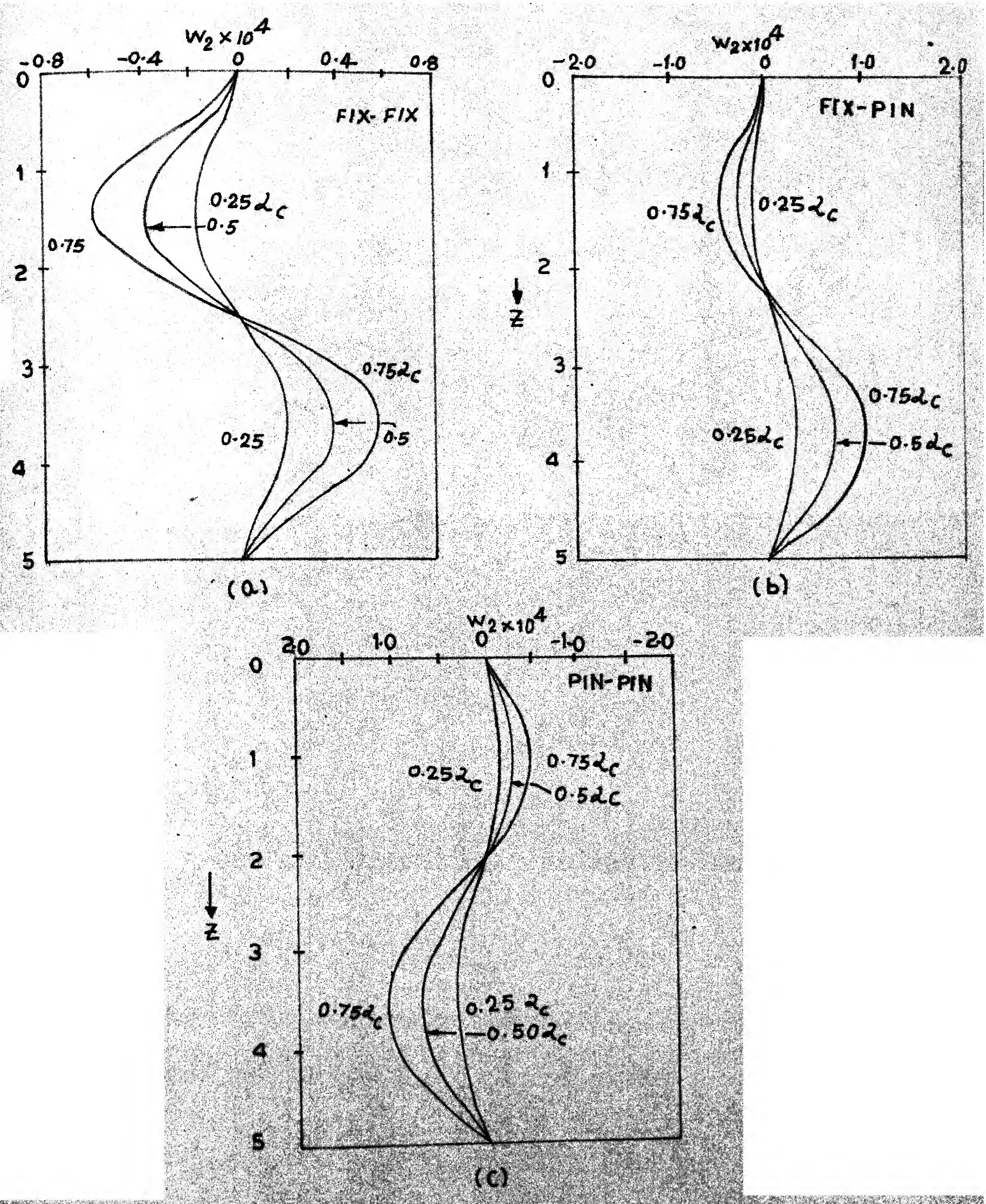


FIG.3.5 EFFECT OF α ON THE VARIATION OF W_2 WITH DEPTH

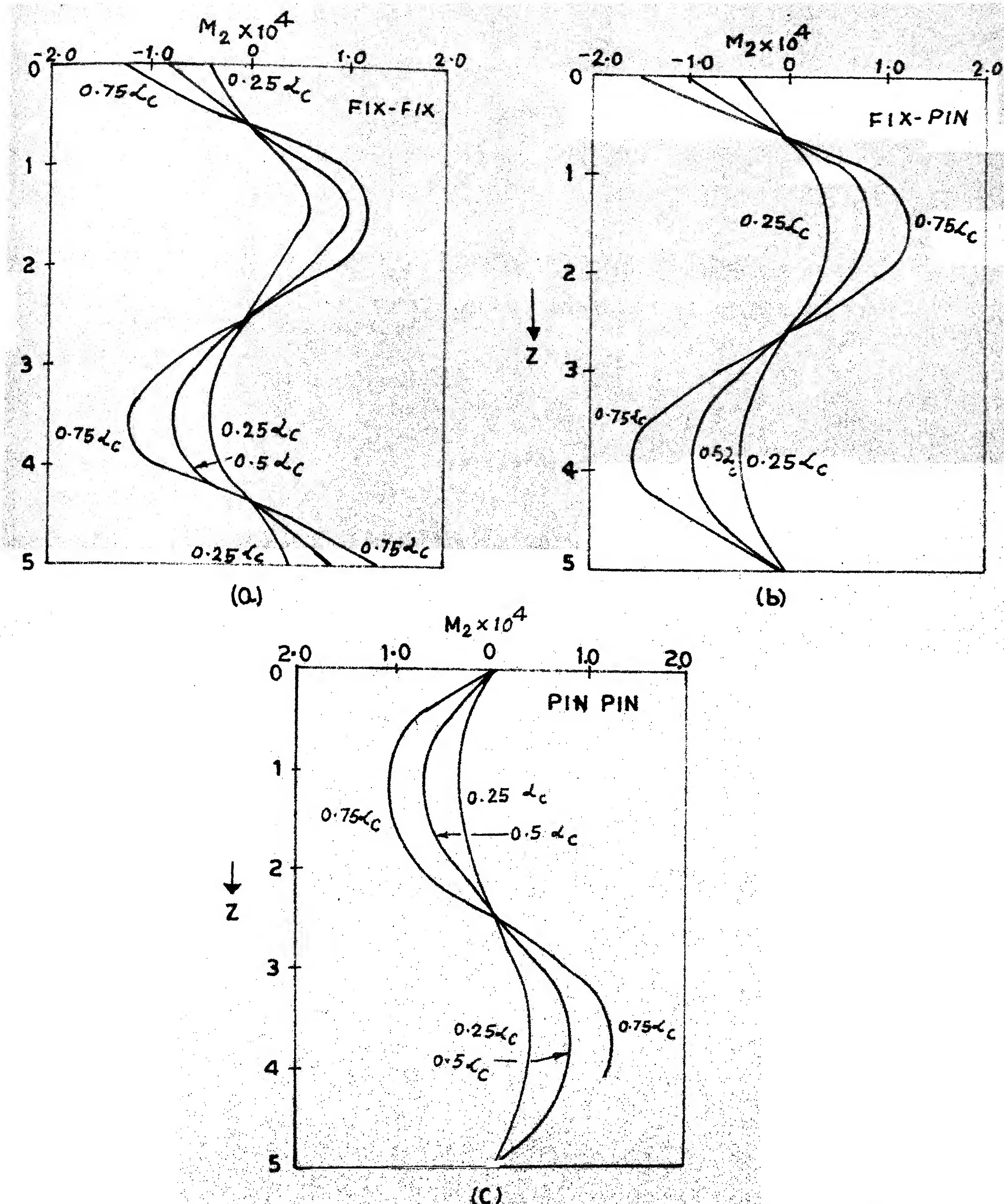


FIG.3.6 EFFECT OF α ON THE VARIATION OF M_2 WITH DEPTH

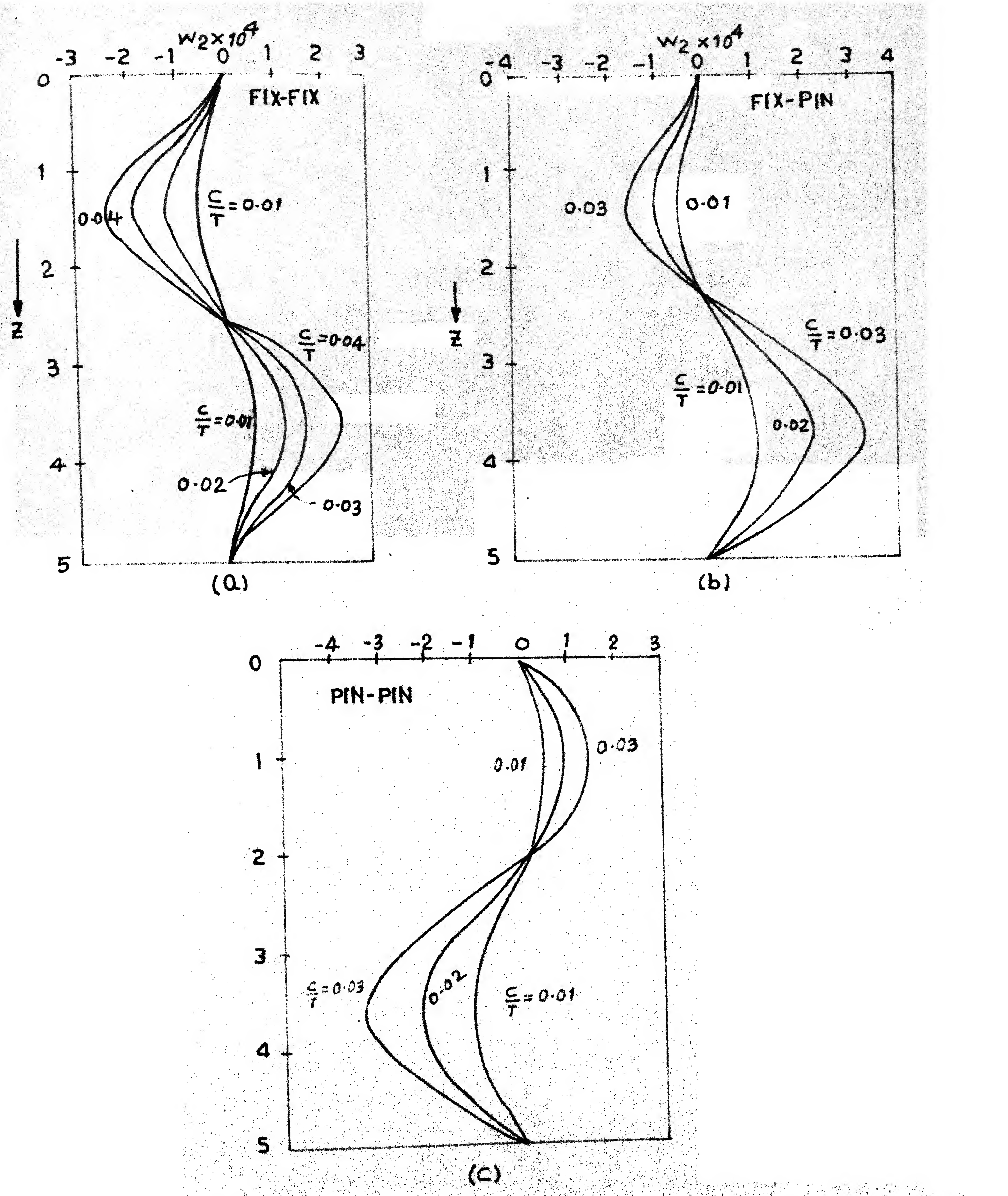
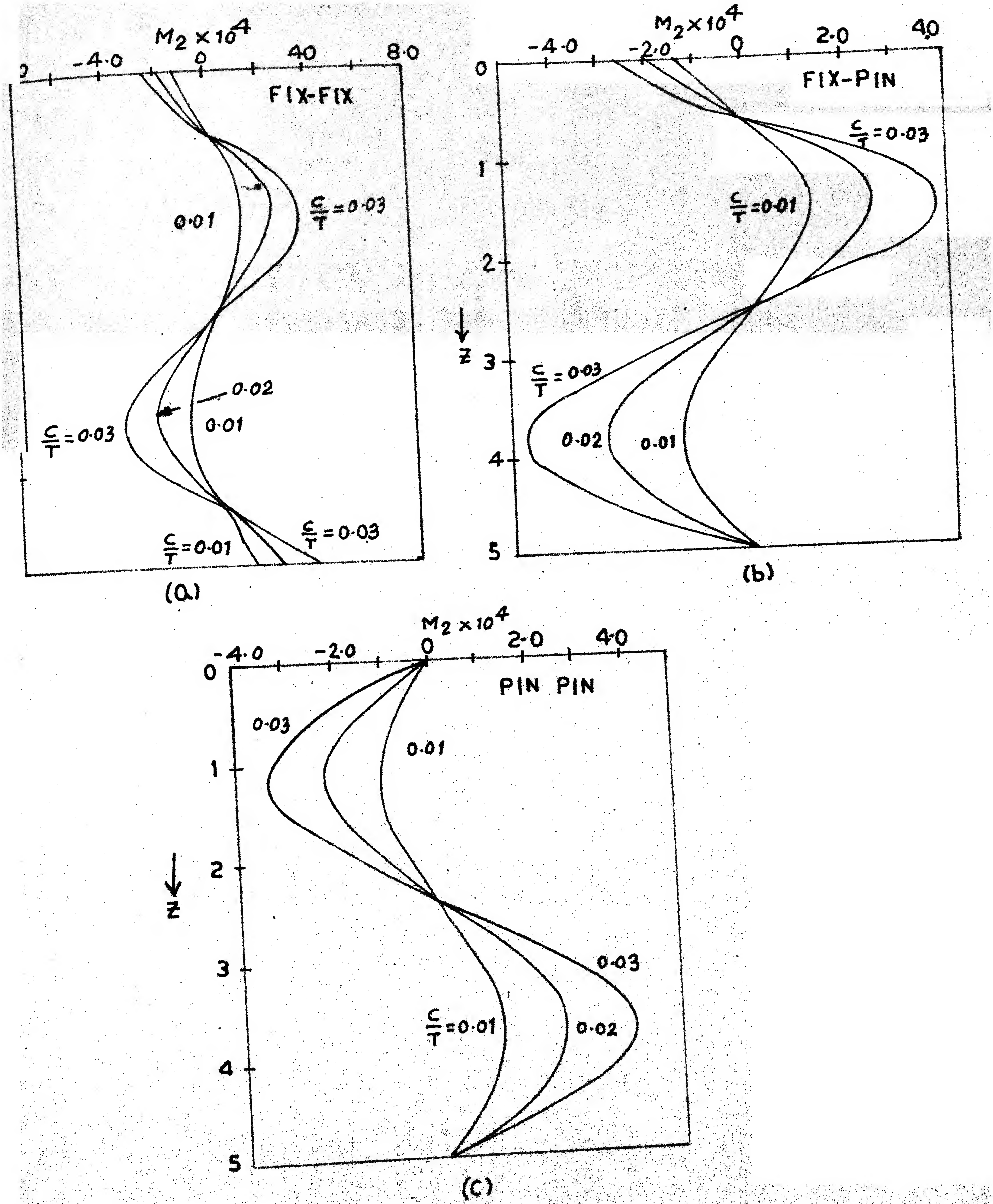


FIG 3 7 EFFECT OF $\frac{C}{T}$ ON THE VARIATION OF W_2 WITH DEPTH



.3·8 EFFECT OF $\frac{C}{T}$ ON THE VARIATION OF M_2 WITH DEPTH

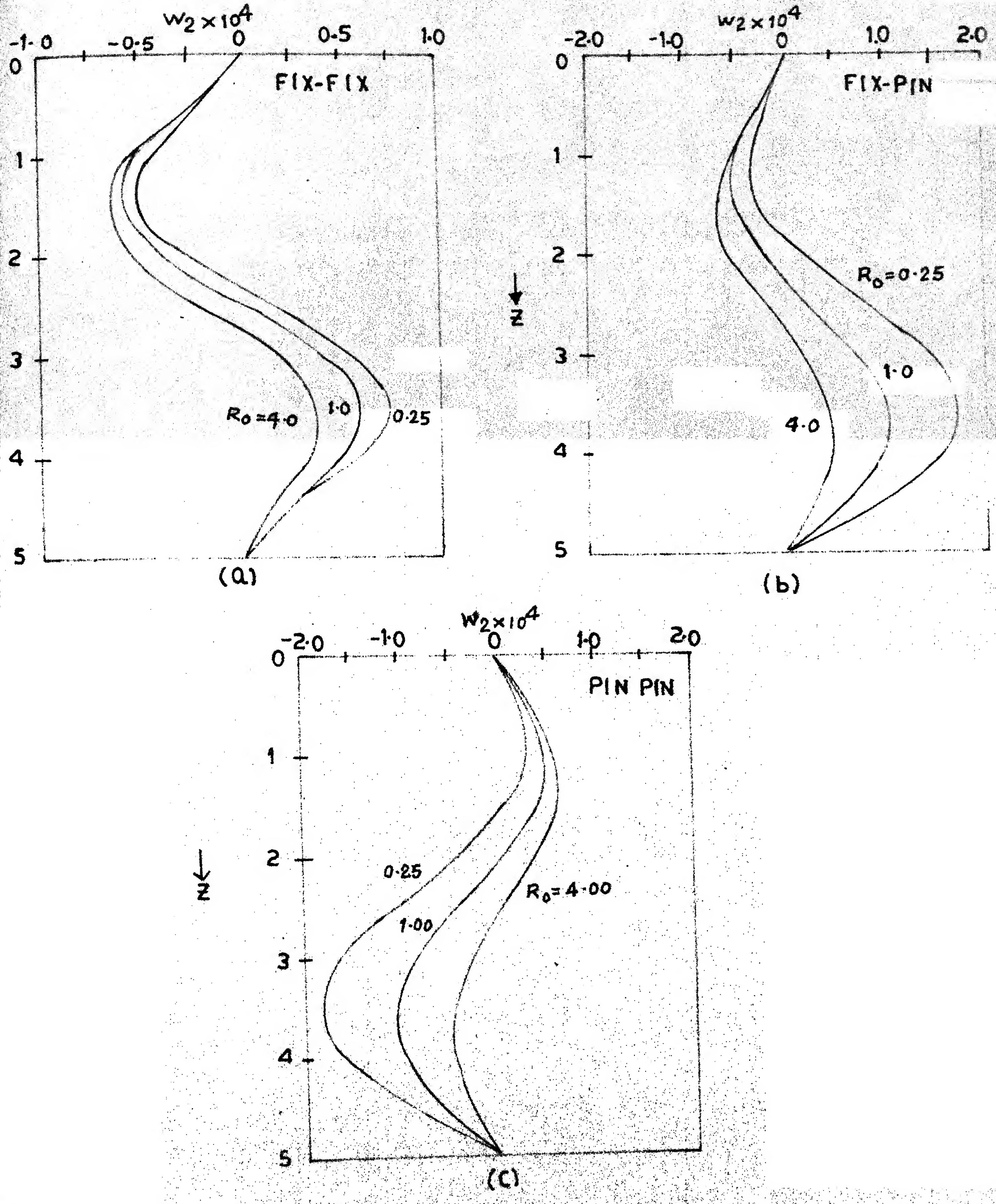


FIG.39 EFFECT OF R_0 ON THE VARIATION OF W_2 WITH DEPTH

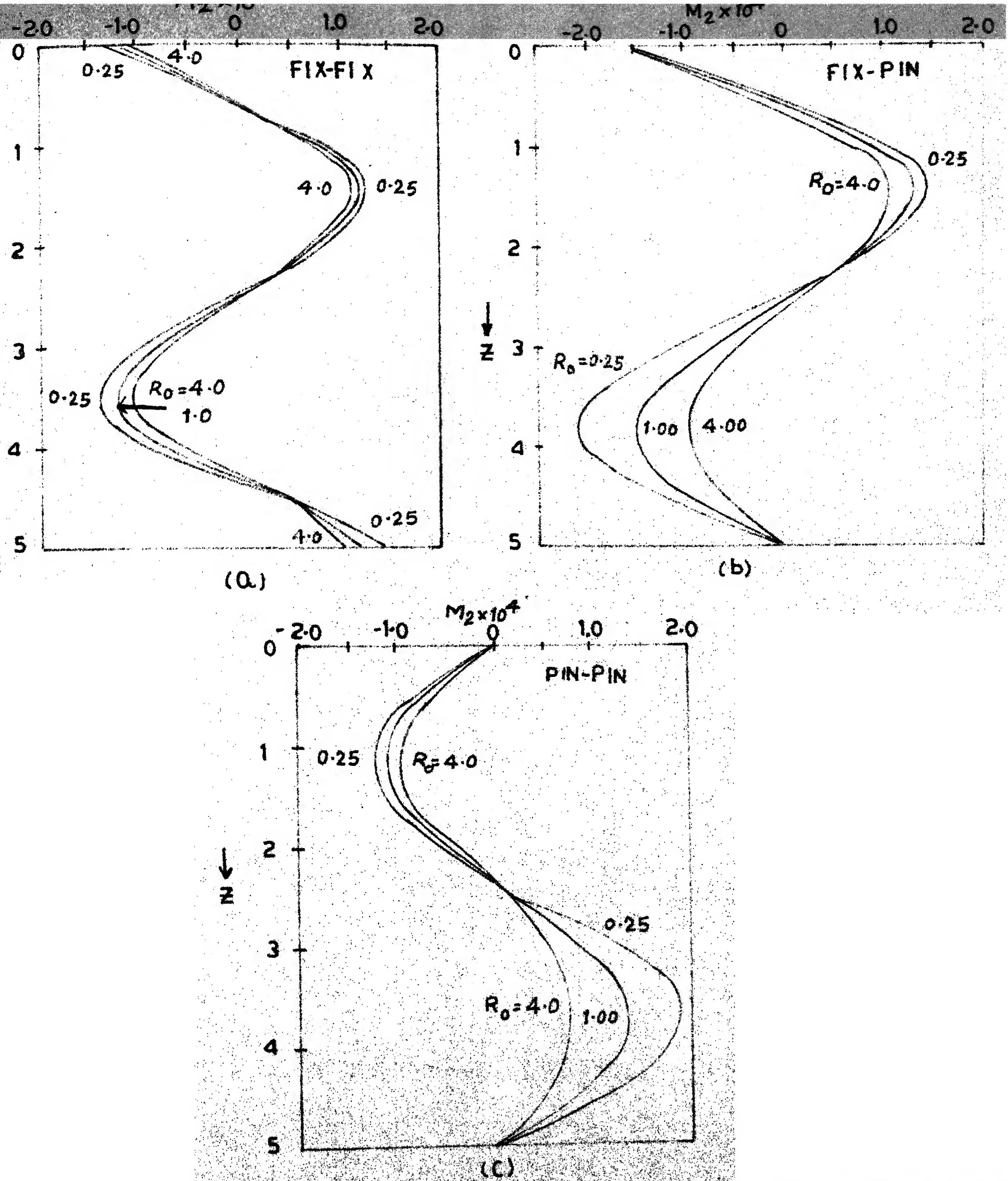


FIG.3·10 EFFECT OF R_o ON THE VARIATION OF M_2 WITH DEPTH

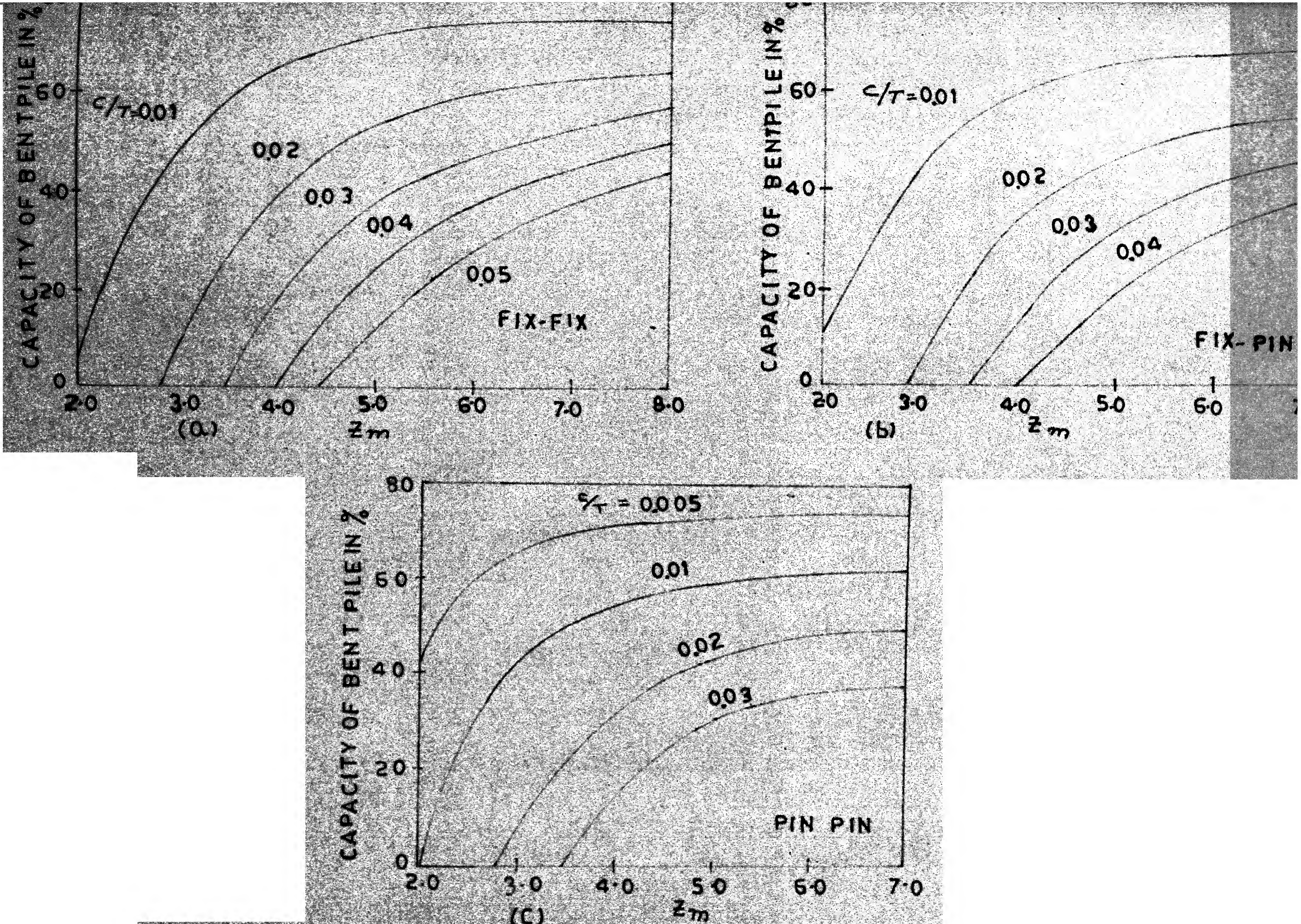


FIG. 3.11 CAPACITY OF BENT PILE Vs Zm

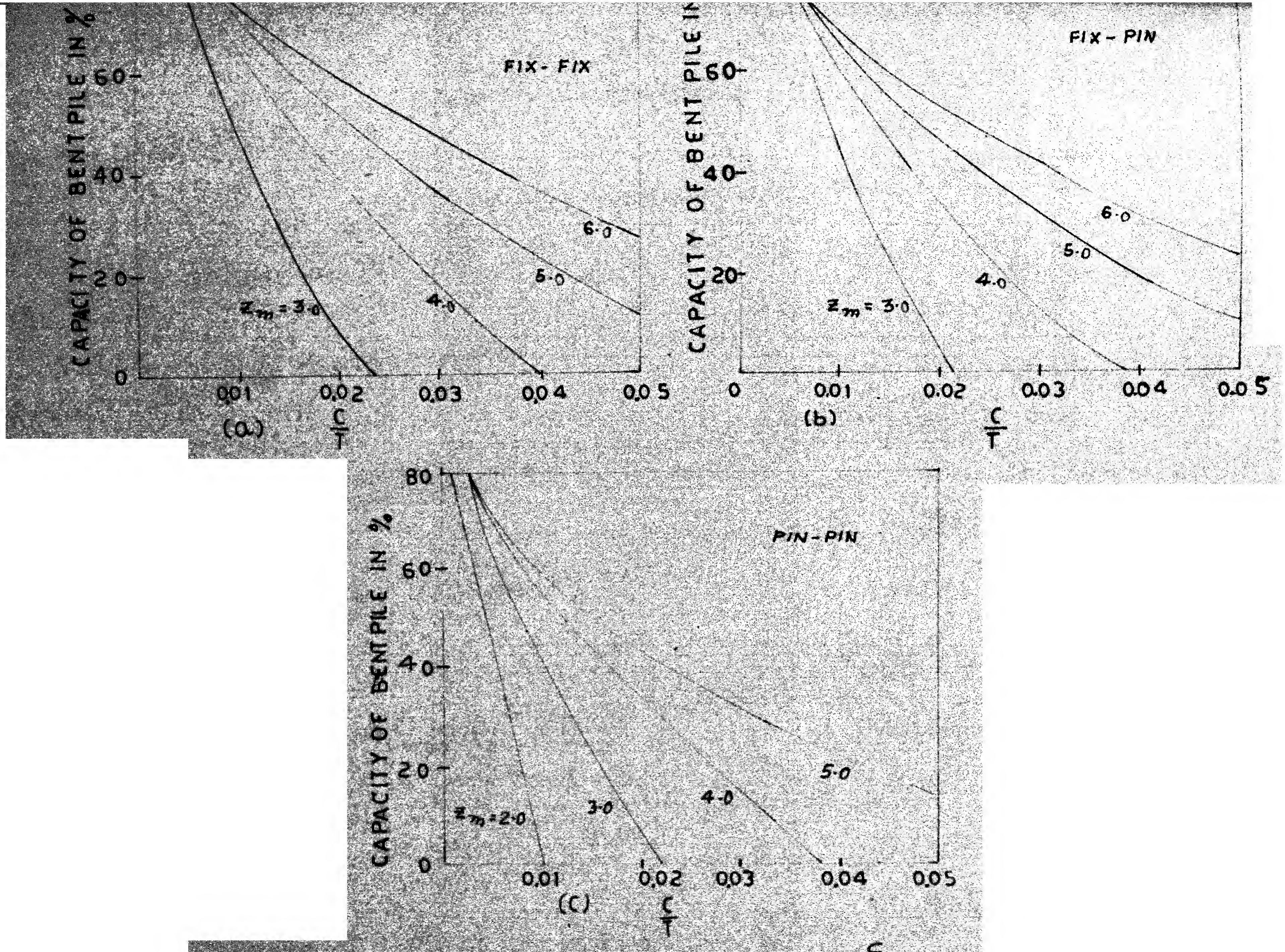


FIG.3.12 CAPACITY OF BENT PILE Vs $\frac{C}{T}$

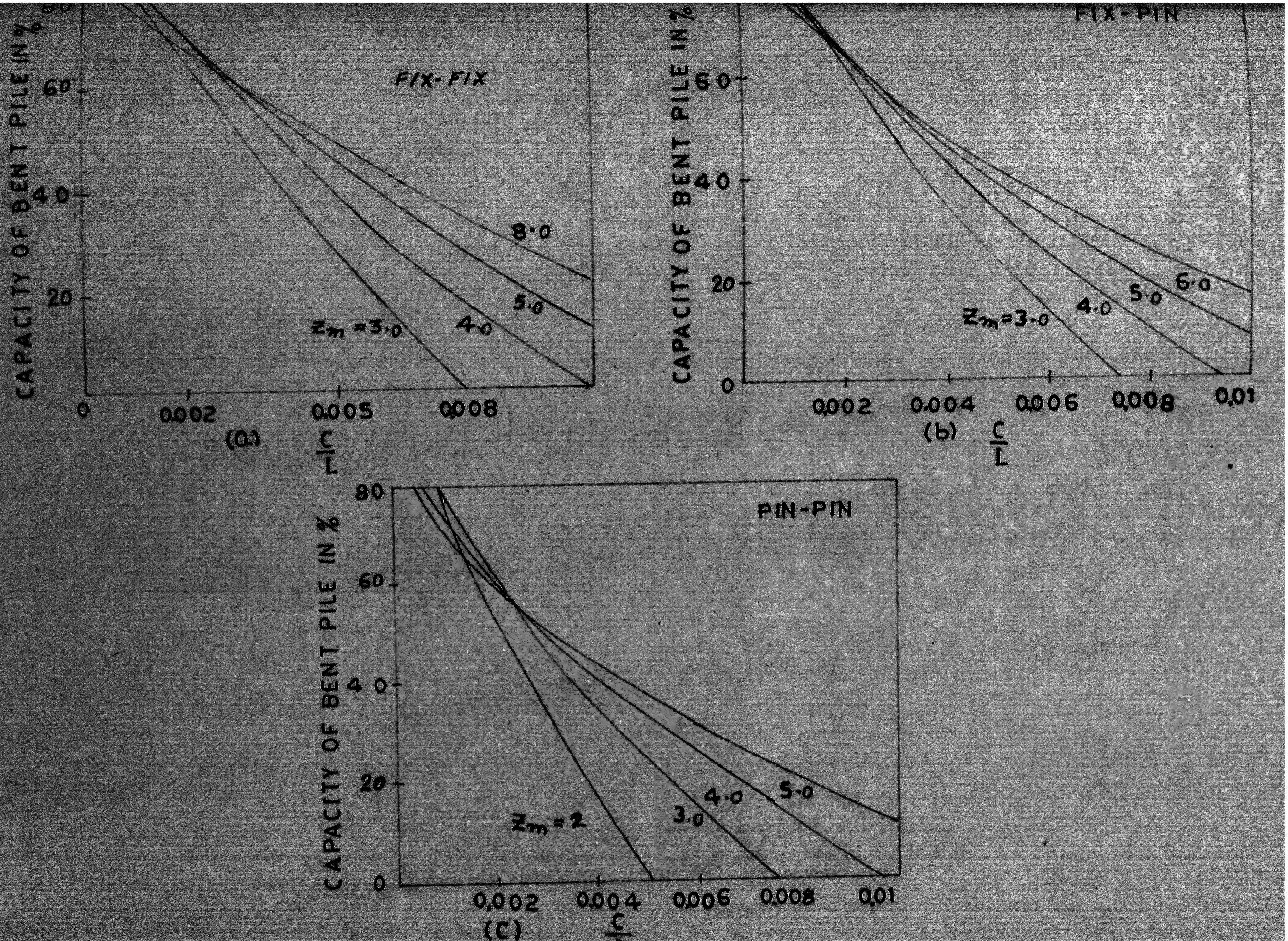


FIG.3·13 CAPACITY OF BENT PILE Vs $\frac{C}{L}$

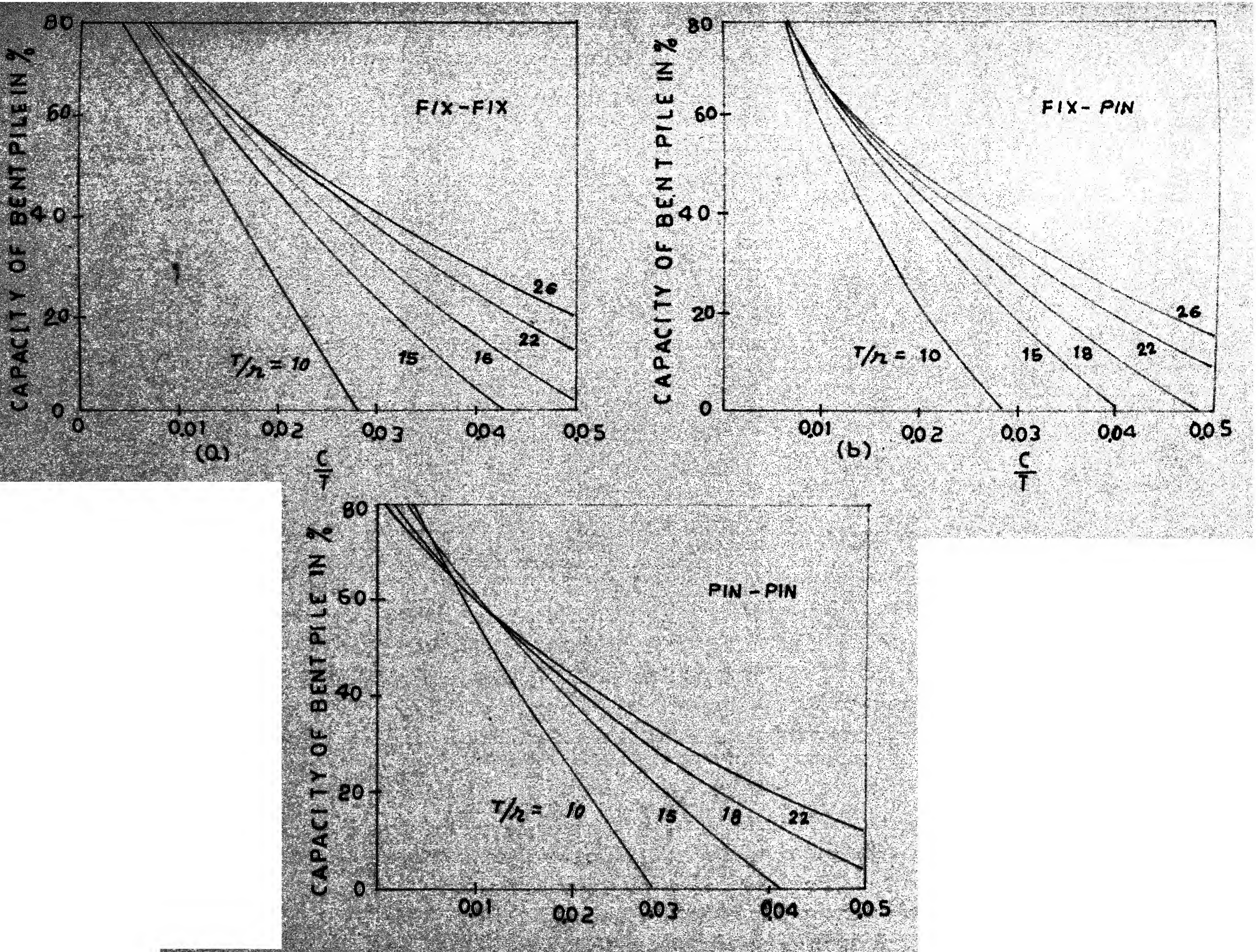


FIG. 3·14 CAPACITY OF BENT PILE Vs $\frac{I}{T}$

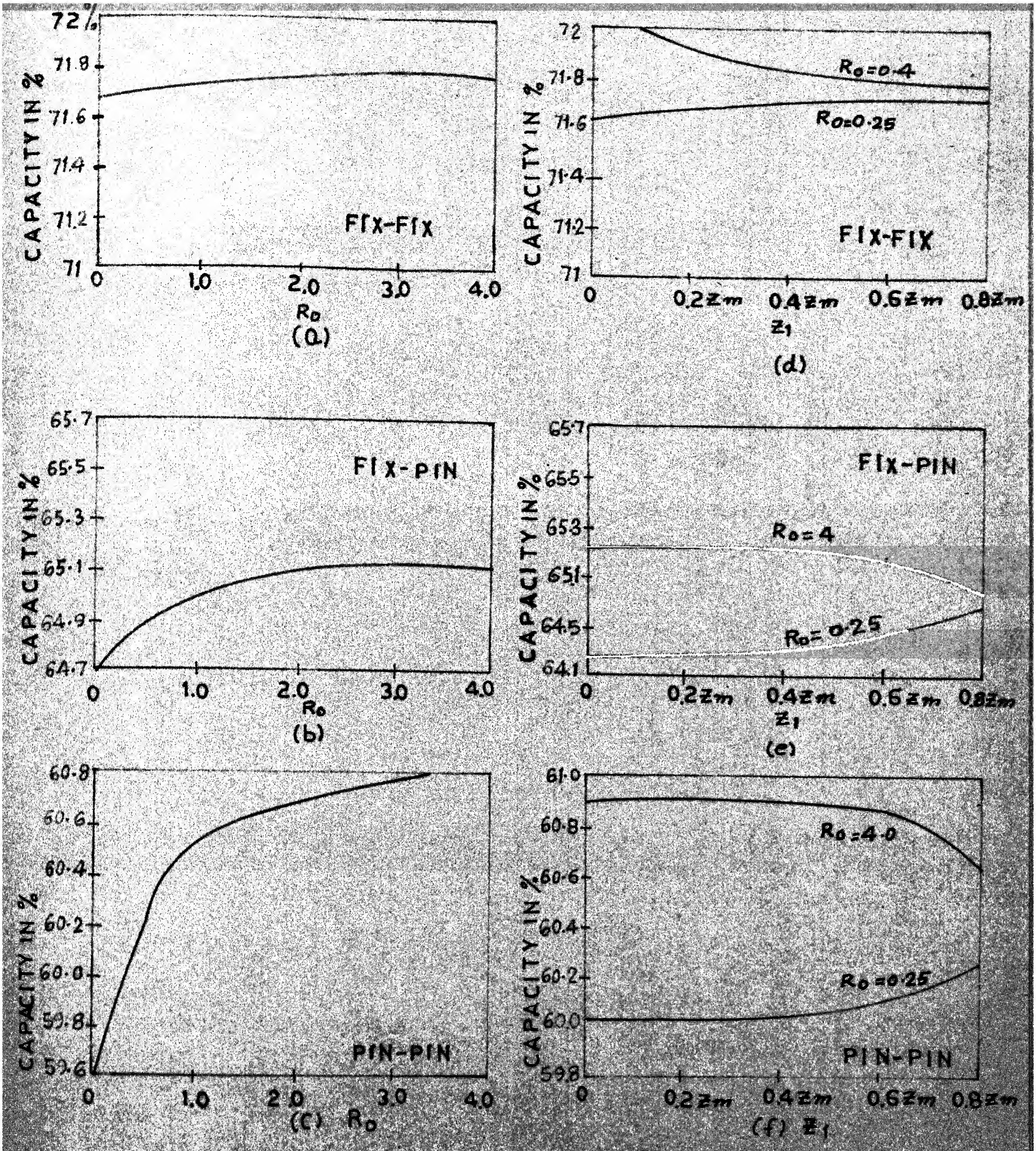


FIG.3.15 EFFECT OF R_o AND Z_1 ON BENT PILE CAPACITY FOR DIFFERENT END CONDITIONS

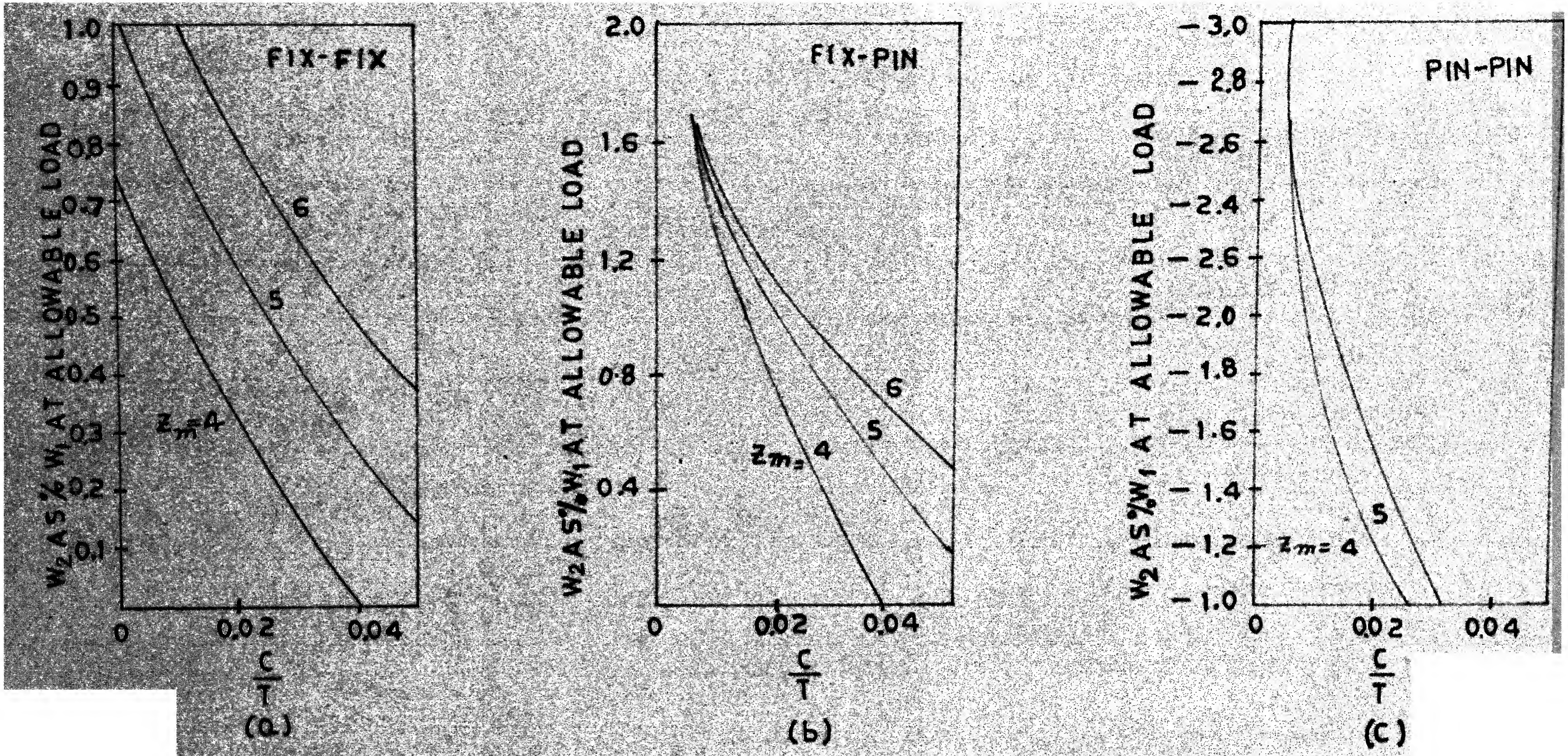


FIG 3.16 PERCENTAGE INCREASE IN DEFLECTION Vs $\frac{C}{T}$

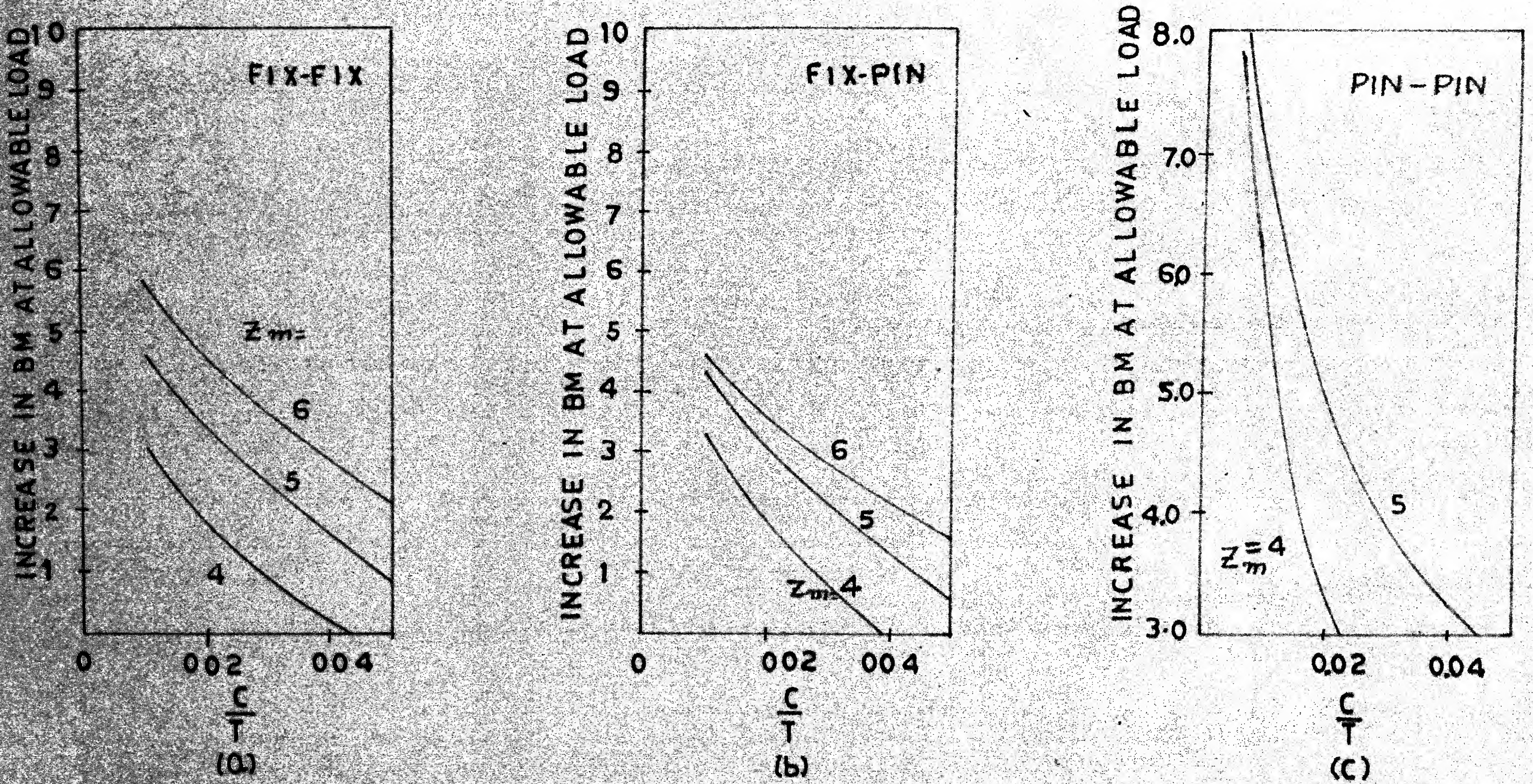


FIG.3.17 PERCENTAGE INCREASE IN BENDING MOMENT Vs $\frac{C}{T}$

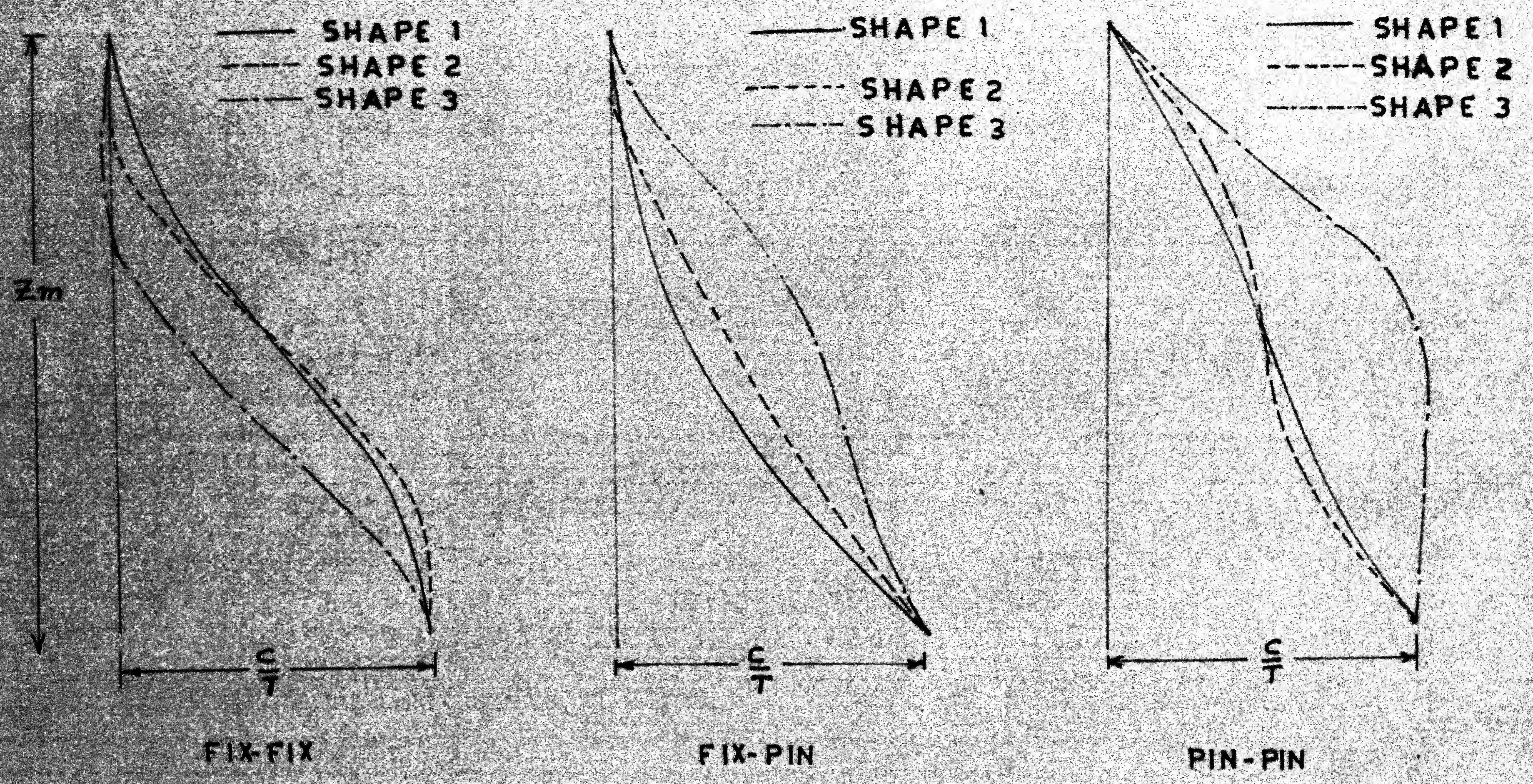


FIG.3·18 TYPICAL SHAPES FOR DIFFERENT END CONDITIONS

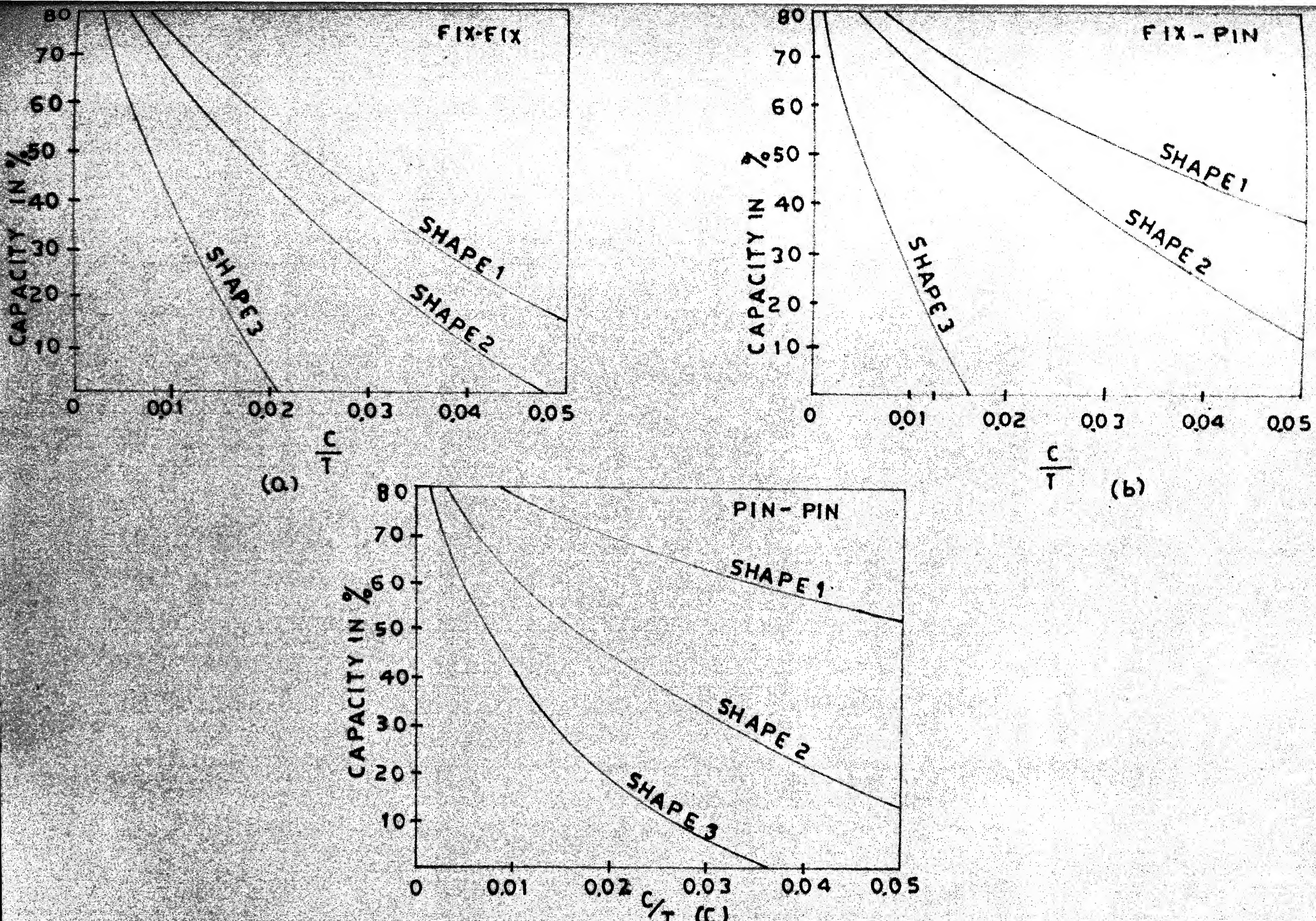


FIG.3·19 LOAD CARRYING CAPACITIES FOR DIFFERENT SHAPES

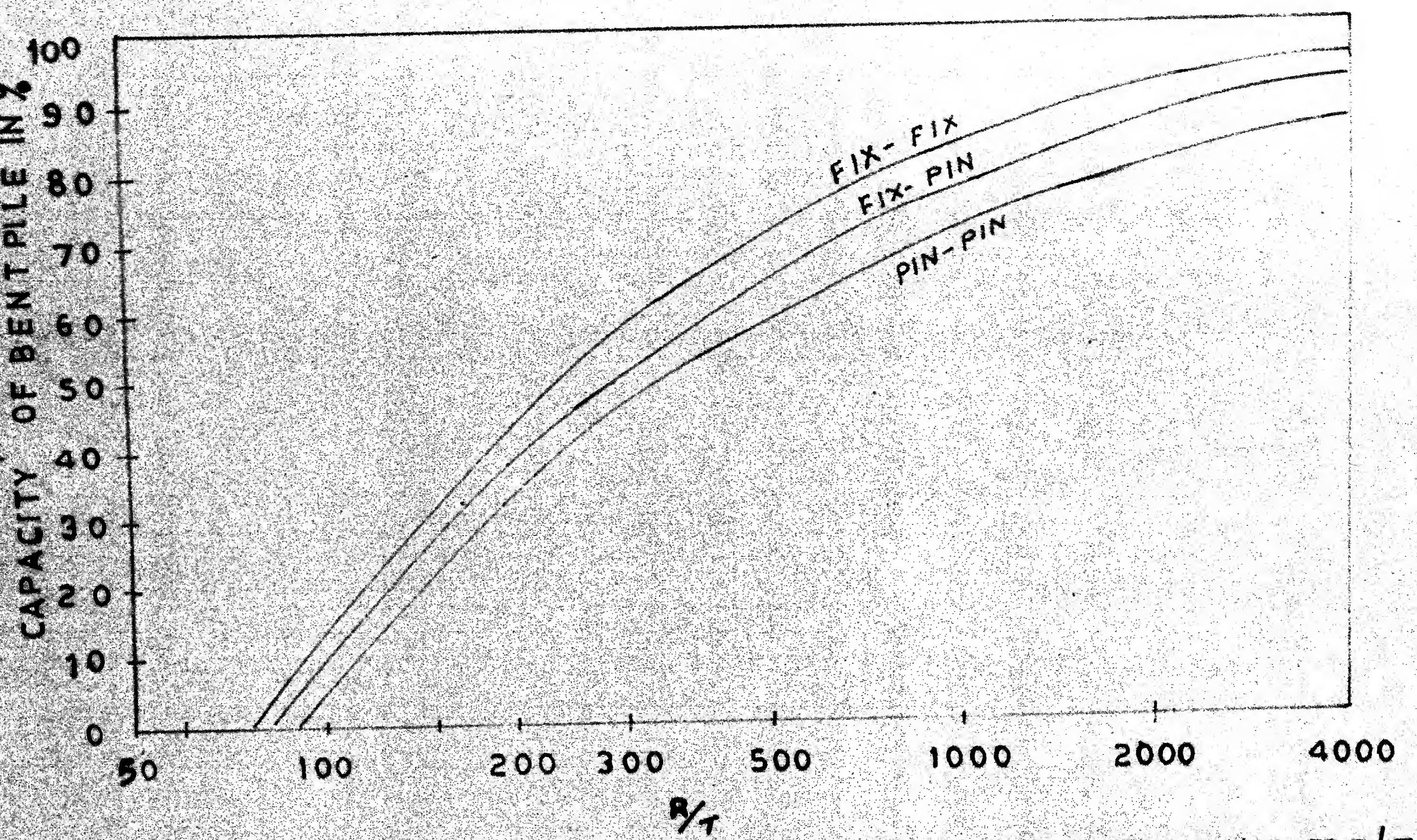


FIG.3·20 BENT PILE CAPACITY FOR DIFFERENT END CONDITIONS W.R.T. R/T

CHAPTER IV

INITIALLY BENT FRICTION PILES IN LAYERED AND NON-HOMOGENEOUS SOILS

4.1 General:

Pile foundations derive their strength both from end bearing resistance and skin friction. In Chapter III, the analysis of end bearing piles has been presented. In this chapter the influence of friction along the pile on the load carrying capacity of the bent pile is studied by modifying the governing differential equation for both cohesive and cohesionless soils. Three sub-soil conditions as shown in Fig. (4.1) are analyzed. (1) Two layer system of cohesive soils, (2) Two layered system of cohesionless soil, (3) non-homogeneous soils with polynomial variation of subgrade reaction. The polynomial variation of subgrade reaction takes into account any deviation of soil modulus from the idealized variations such as soil modulus constant with depth and linearly varying with depth (Matlock and Reese, 1960). The effect of down drag is also studied. Because a closed form solution is difficult to obtain for the above mentioned models finite difference technique is used based on the method suggested by Glesser (1953).

In the case of cohesive soils, the coefficient of subgrade reaction is assumed to be invariant with depth for each layer (Terzaghi,1955) and thus the total load taken by the skin friction is directly proportional to the surface area. This will provide a linear distribution of axial load in the pile. In the case of cohesionless soils the coefficient of subgrade reaction is assumed to be linearly varying with depth (Terzaghi, 1955) and since the skin friction is a linear function of depth, the axial force distribution becomes parabolic. In the case of polynomial variation of coefficient of subgrade reaction also the axial load distribution is assumed to be parabolic.

4.2 Finite Difference Technique:

4.2.1 Basic Concept:

In this method the continuous bent pile is divided into N segments. The unknown function, for the present case additional deflection, is replaced by N algebraic variables at the discrete points. Consequently, the differential equations which describe the behaviour of continuous initially bent piles are replaced by N simultaneous algebraic equations in these variables. In the process the derivative of the unknown function at the point is approximated by an algebraic expression consisting of the value of the function at that point and several near-by points. Hence, the closer the points are to one another the better is the agreement between the derivative

and its algebraic approximation and the more accurate will be the solution. However, as the number of points increases so does the number of simultaneous equations that must be solved.

4.2.2 Solution Procedure:

The pile is divided into n segments. In addition to this two segments at the top of the pile and two segments at the bottom are imagined as shown in Fig. 4.2. The following relationships can be written about node i in difference form.

$$\left(\frac{dw_2}{dz}\right)_i = \frac{(w_2)_{i+1} - (w_2)_{i-1}}{2h}$$

$$\left(\frac{d^2w_2}{dz^2}\right)_i = \frac{(w_2)_{i+1} - 2(w_2)_i + (w_2)_{i-1}}{h^2}$$

$$\left(\frac{d^4w_2}{dz^4}\right)_i = \frac{(w_2)_{i+2} - 4(w_2)_{i+1} + 6(w_2)_i - 4(w_2)_{i-1} + (w_2)_{i-2}}{h^4} \qquad (4.1)$$

where,

h = increment length

$(w_2)_i$ = the additional deflection at node i.

Substitution of the above differentials in the governing differential equation results an algebraic equation with the $(w_2)_{i+2}$, $(w_2)_{i+1}$, $(w_2)_i$, $(w_2)_{i-1}$ and $(w_2)_{i-2}$ as variables. This type of equation can be obtained for all nodes (0 to n) which forms n + 1 equations in n + 5 unknowns.

Therefore, four boundary conditions are needed to get the complete solution. Two boundary conditions at top and two boundary conditions at bottom for fix-fix, fix-pin, pin-pin are provided as discussed in previous chapter.

A convenient method of solving these large number of simultaneous equations has been suggested by Glesser (1953). The steps in Glessers method are as follows.

1. A recursive equation is set-up for deflections at each point along the pile, starting at the top.

2. The coefficients in the recursive equation are computed along the length of the pile.

3. The deflection at the bottom of the pile is calculated from recursive equation and the boundary conditions at the bottom of the piles.

4. The deflections for all upper points, starting at the point next to the bottom and progressing upward, can then be computed using the recursive equation.

4.3 Differential Equation for Initially Bent Friction Pile:

The governing differential equation for a friction bent pile is given,by,

$$EI \frac{d^4u_2}{dy^4} + \frac{d}{dy}\left[P(y)\frac{du}{dy}\right] + Ky(y) = 0 \qquad (4.2)$$

where, $u = u_1 + u_2$ and u_1 and u_2 are initial and additional deflections respectively.

The solution of Eqn. (4.2) depends upon the form of variation of axial force $P_{(y)}$ and also on the variation of Soil modulus $K_{(y)}$. The most simple case for which closed form solution is available is that when the axial force as well as the soil modulus are invariant with depth. The analysis of the bent piles with friction is presented for the cases of cohesive and cohesionless soils.

Since the finite difference method is applicable only for continuous functions, the discontinuity in the soil modulus at the interface of two layers is approximated as shown in Fig. 4.3.

4.4 Analysis of Bent Friction Piles in Cohesive Soil:

Assuming the soil modulus to be invariant with depth in each layer and axial load in pile to be linearly decreasing with depth Eqn. (4.2) reduces to,

$$EI \frac{d^4u_{2t}}{dy^4} + P\left(1 - \varphi_1 \frac{y}{L}\right) \frac{d^2u_{2t}}{dy^2} - \frac{P\varphi_1}{L} \frac{du_{2t}}{dy} + K_t\, u_{2t}$$

$$= -P\left(1-\varphi_1 \frac{y}{L}\right) \frac{d^2u_1}{dy^2} + P \frac{\varphi_1}{L} \frac{du_1}{dy}$$

$$\text{for } 0 \leq y \leq L_1 \qquad (4.3)$$

$$EI \frac{d^4u_{2b}}{dy^4} + P\left(1- \varphi_1 \frac{L_1}{L} - \varphi_2 \frac{y-L_1}{L}\right) \frac{d^2u_{2b}}{dy^2} - P \frac{\varphi_2}{L} \frac{du_{2b}}{dy}$$

$$+ K_b u_{2b} = -P\left(1- \varphi_1 \frac{L_1}{L} - \varphi_2 \frac{y-L_1}{L}\right) \frac{d^2u_1}{dy^2}$$

$$+ P \frac{\varphi_2}{L} \frac{du_1}{dy} \quad \text{for } L_1 \leq y \leq L \qquad (4.4)$$

where φ_1 and φ_2 are the skin friction factor in the top layer and the bottom layer respectively. All other terms are explained in the previous chapter.

The relative stiffness factor T is defined as $T = [\frac{EI}{Kt}]^{1/4}$

Non-dimensionalising deflections and length by using T as

Non-dimensionalized depth coefficients $z = y/T$

Non-dimensionalized length of pile $z_m = L/T$

Non-dimensionalized thickness of upper layer $z_1 = \frac{L_1}{T}$

Non-dimensionalized pile offset $= \frac{C}{T}$

Non-dimensionalized initial deflections $w_1 = \frac{u_1}{T}$

Non-dimensionalized additional deflections $w_2 = \frac{u_2}{T}$

Non-dimensionalized total deflections $w = \frac{u}{T}$

Substituting the above non-dimensional parameters in the Eqns. (4.3) and (4.4) they can be written in non-dimensional form as,

$$\frac{d^4 w_{2t}}{dz^4} + \alpha\ (1-\beta_1 z)\ \frac{d^2 w_{2t}}{dz^2} - \alpha\beta_1 \frac{dw_{2t}}{dz} + w_{2t}$$

$$= -\alpha(1-\beta_1 z)\ \frac{d^2 w_1}{dz^2} + \alpha\beta_1 \frac{dw_1}{dz} \quad \text{for } 0 \leq z \leq z_1 \qquad (4.5)$$

$$\frac{d^4 w_{2b}}{dz^4} + \alpha(1-\beta_1 z_1-\beta_2(z-z_1))\ \frac{d^2 w_{2b}}{dz^2} - \alpha\beta_2 \frac{dw_{2b}}{dz} + R_o w_{2b}$$

$$= -\alpha(1-\beta_1 z_1-\beta_2(z-z_1))\ \frac{d^2 w_1}{dz^2} + \alpha\beta_2 \frac{dw_1}{dz}$$

$$\text{for } z_1 \leq z \leq z_m \qquad (4.6)$$

where,

$$\beta_1 = \frac{\varphi_1}{z_m}, \qquad \beta_2 = \frac{\varphi_2}{z_m}, \qquad \text{and} \quad R_o = \frac{K_b}{K_t}$$

The remaining terms are explained previously.

For convenience the total pile length is divided into n elements such that a node falls at the interface of two layers. The node n_1 at the interface of two layers is given by $n_1 = \frac{z_1}{z_m} \times n$

The length of pile increment is given by $h = \frac{z_m}{n}$.

Since the initial shape of the bent pile w_1 is an assumed polynomial it can be differentiated and substituted in Eqn. (4.5) and (4.6). At any general node i the right hand side of the differential equations (4.5) and (4.6) can be written as,

$$RHS'(I) = -\alpha(1-\beta_1 z(I)) \left(\frac{d^2w_1}{dz^2}\right)_{z=z(I)} + \alpha\beta_1 \left(\frac{dw_1}{dz}\right)_{z=z(I)}$$

$$\text{for } 0 \leq i \leq n_1 \tag{4.7}$$

$$RHS'(I) = -\alpha(1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) \left(\frac{d^2w_1}{dz^2}\right)_{z=z(I)}$$

$$+ \alpha\beta_2 \left(\frac{dw_1}{dz}\right)_{z=z(I)} \quad \text{for } n_1 \leq i \leq n \tag{4.8}$$

where z(I) is the depth of the node i which can be obtained as, z(I) = (i) (h).

Substituting the difference equations (4.1) in Eqns. (4.5) and (4.6) the left hand side of these equations at any node i can be written as,

$$\frac{1}{2h^4}\,[2W2(I+2) + FA(I)\ W2(I+1) + FB(I)\ W2(I) + FC(I)\ W2(I-1) + 2W2(I-2)] \qquad (4.9)$$

where W2(I) is the additional deflection, w_2 at node i,

FA(I), FB(I), FC(I) are the coefficients of W2(I+1), W2(I), and W2(I-1) respectively which can be obtained as below:

$$FA(I) = -8.0 + 2\alpha h^2\ (1-\beta_1 z(I)) - \alpha\beta_1 h^3$$

$$FB(I) = 12.0 - 4\alpha h^2\ (1-\beta_1 z(I)) + 2h^4$$

$$FC(I) = -8.0 + 2\alpha h^2\ (1-\beta_1\ z(I)) + \alpha\beta_1 h^3$$

$$\text{for } 0 \leq i \leq n_1-5$$

$$FA(I) = -8.0 + 2\alpha h^2\ (1-\beta_1 z(I)) - \alpha\beta_1 h^3$$

$$FB(I) = 12.0 - 4\alpha h^2\ (1-\beta_1 z(I)) + 2h^4\ (1+\frac{R_o-1}{10}\ (i+5-n_1))$$

$$FC(I) = -8.0 + 2\alpha h^2\ (1-\beta_1 z(I)) + \alpha\beta_1 h^3$$

$$\text{for } n_1-5 \leq i \leq n_1$$

$$FA(I) = -8.0 + 2\alpha h^2\ (1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) - \alpha\beta_2 h^3$$

$$FB(I) = 12.0 - 4\alpha h^2\ (1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) + 2h^4\ (1+\frac{R_o-1}{10}\ (i+5-n_1))$$

$$FC(I) = -8.0 + 2\alpha h^2\ (1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) + \alpha\beta_2 h^3$$

$$\text{for } n_1 \leq i \leq n_1 + 5$$

$$FA(I) = -8.0 + 2\alpha h^2 (1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) - \alpha\beta_2 h^3$$

$$FB(I) = 12.0 - 4\alpha h^2 (1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) + 2h^4 R_c$$

$$FC(I) = -8.0 + 2\alpha h^2 (1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) + \alpha\beta_2 h^3$$

for $n_1+5 \leq i \leq n$

Defining,

$$RHS(I) = 2h^4 \, RHS'(I) \tag{4.10}$$

the differential equations (4.5) and (4.6) at any node i can be written in terms of the additional deflections of neighbouring nodes as,

$$2W2(I+2) + FA(I)\,W2(I+1) + FB(I)\,W2(I) + FC(I)\,W2(I-1) + 2W2(I-2) = RHS(I) \tag{4.11}$$

4.4.1 Fix-Fix End Conditions:

The initial shape satisfying both natural and geometric boundary conditions for fix-fix end conditions is given by,

$$w_1 = \frac{C}{T}\left[1.25 \frac{z^2}{z_m^2} + 5.0 \frac{z^3}{z_m^3} - 8.75 \frac{z^4}{z_m^4} + 3.5 \frac{z^5}{z_m^5}\right] \tag{3.20}$$

Differentiating Eqns. (3.20) and substituting in Eqn. (4.7) and (4.8) gives,

$$RHS'(I) = -\alpha(1-\beta_1 z(I)) \frac{C}{T}\left[\frac{2.5}{z_m^2} + 30 \frac{z(I)}{z_m^3} - 105 \frac{z(I)^2}{z_m^4} + 70 \frac{z(I)^3}{z_m^5}\right] + \alpha\beta_1 \frac{C}{T}\left[2.5 \frac{z(I)}{z_m^2} + 10 \frac{z(I)^2}{z_m^3} - 35 \frac{z(I)^3}{z_m^4} + 17{,}5 \frac{z(I)^4}{z_m^5}\right] \text{ for } 0 \leq i \leq n_1 \tag{4.12}$$

$$RHS'(I) = -\alpha(1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I)) \frac{C}{T} \left[\frac{2.5}{z_m^3} + 30 \frac{z(I)}{z_m^3} - 105 \frac{z(I)^2}{z_m^4} + 70 \frac{z(I)^3}{z_m^5} \right]$$

$$+ \alpha\beta_2 \frac{C}{T} \left[2.5 \frac{z(I)}{z_m^2} + 10 \frac{z(I)^2}{z_m^3} - 35 \frac{z(I)^3}{z_m^4} + 17.5 \frac{z(I)^4}{z_m^5} \right] \quad \text{for } n_1 \leq i \leq n \qquad (4.13)$$

$$RHS\ (I) = 2h^4\ RHS'(I)$$

Boundary conditions at top:

At $z = 0$, $\quad w_2 = 0 \quad$ and $\quad \frac{dw_2}{dz} = 0$

Writing these boundary conditions in finite difference form,

$$\frac{W2(1) - W2(-1)}{2h} = 0$$

gives, $\quad W2(1) = W2(-1)$

Writing the governing differential equation in finite difference form Eq. (4.11) at node 1, and substituting top boundary conditions W2(0) = 0 and W2(1) = W2(-1) leads to,

$$W2(1) = \frac{-FA(1)}{2.0 + FB(1)} W2(2) + \frac{-2}{2.0 + FB(1)} W2(3) + \frac{RHS(1)}{2.0+FB(1)}$$

$$= A(1)\ W2(2) + B(1)\ W2(3) + C(1)$$

Similarly writing the governing differential equations in finite difference form Eq. (4.11) at node 2 and substituting W2(0), W2(1) leads to

$$W2(2) = A(2)\ W2(3) + B(2)\ W2(4) + C(2)$$

where,

$$A(2) = -\frac{[FA(2) + B(1)\ FC(2)]}{FB(2) + A(1)\ FC(2)}$$

$$B(2) = \frac{-2.0}{FB(2) + A(1)\ FC(2)}$$

$$C(2) = \frac{RHS(2) - C(1)\ FC(2)}{FB(2) + A(1)\ FC(2)}$$

For any general node i, the recurrence relation can be written as,

$$W2(I) = A(I)\ W2(I+1) + B(I)\ W2(I+2) + C(I) \qquad (4.14)$$

where,

$$A(I) = -\frac{[FA(I) + B(I-1)\ FC(I) + 2A(I-2)\ B(I-1)]}{FB(I) + A(I-1)\ FC(I) + 2A(I-1)\ A(I-2) + 2B(I-2)} \qquad (4.15)$$

$$B(I) = \frac{-2.0}{FB(I) + A(I-1)\ FC(I) + 2A(I-1)\ A(I-2) + 2B(I-2)} \qquad (4.16)$$

$$C(I) = \frac{RHS(I) - C(I-1)\ FC(I) - 2A(I-2)\ C(I-1) - 2C(I-2)}{FB(I) + A(I-1)\ FC(I) + 2A(I-1)\ A(I-2) + 2B(I-2)} \qquad (4.17)$$

Writing the boundary conditions at the bottom end of the pile in finite difference form,

$$W2(n) = 0, \qquad \left(\frac{dw_2}{dz}\right)_{z=z_m} = \frac{W2(n+1) - W2(n-1)}{2h} = 0$$

leads to W2(n+1) = W2(n-1).

Writing the general recurrence relationship Eqn.(4.14) at node (n-1) and substituting W2(n) = 0 and W2(n+1) = W2(n-1) the deflection at node (n-1) can be obtained as,

$$W2(n-1) = \frac{C(n-1)}{1-B(n-1)}$$

Knowing W2(n-1), W2(n) and the recurrence coefficients A(I), B(I), C(I) at all nodes, by the process of back substitution, additional deflections W2(I) at all nodes can be evaluated.

4.4.2 Fix-Pin End Conditions:

The initial shape satisfying natural and geometric boundary conditions corresponding to fix-pin end conditions is given by,

$$w_1 = \frac{C}{T}\left[0.5\frac{z^2}{z_m^2} + 7.5\frac{z^3}{z_m^3} - 11.75\frac{z^4}{z_m^4} + 4.75\frac{z^5}{z_m^5}\right] \tag{3.29}$$

Differentiating Eqn. (3.29) and substituting in Eqn. (4.7) and (4.8) gives,

$$\begin{aligned} RHS'(I) = & -\alpha(1-\beta_1 z(I))\frac{C}{T}\left[\frac{1}{z_m^2} + 45\frac{z(I)}{z_m^3} - 141\frac{z(I)^2}{z_m^4}\right. \\ & \left. + 95\frac{z(I)^3}{z_m^5}\right] + \alpha\beta_1\frac{C}{T}\left[\frac{z(I)}{z_m^2} + 22.5\frac{z(I)^2}{z_m^3}\right. \\ & \left. - 47\frac{z(I)^3}{z_m^4} + 23.75\frac{z(I)^4}{z_m^5}\right] \quad \text{for } 0 \le i \le n_1 \end{aligned} \tag{4.18}$$

$$RHS'(I) = -\alpha\,(1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I))\,\frac{C}{T}\,\Big[\,\frac{1}{z_m^{\,2}} + 45\,\frac{z(I)}{z_m^{\,3}} - 141\,\frac{z(I)^2}{z_m^{\,4}} + 95\,\frac{z(I)^3}{z_m^{\,5}}\,\Big]$$

$$+ \alpha\beta_2\,\frac{C}{T}\,\Big[\,\frac{z(I)}{z_m^{\,2}} + 22.5\,\frac{z(I)^2}{z_m^{\,3}} - 47\,\frac{z(I)^3}{z_m^{\,4}} + 23.75\,\frac{z(I)^4}{z_m^{\,5}}\,\Big] \qquad \text{for } n_1 \leq i \leq n \quad (4.19)$$

$$RHS\,(I) = 2h^4\,RHS'(I)$$

Writing Eqn.(4.11) at node 1 and substituting the top boundary conditions expressed in finite difference form, A(1), B(1) and C(1) can be obtained. Similarly writing Eqn. (4.11) at node 2 and substituting W2(0) and W2(1) leads to the evaluation of A(2), B(2) and C(2). The remaining recurrence coefficients A(I), B(I), C(I) for all nodes can be obtained using Eq. (4.15), (4.16) and (4.17).

Expressing the bottom boundary conditions, W2(n) = 0 and $\left(\frac{d^2w2}{dz^2}\right)_{z=z_m} = 0$, in finite difference form and substituting in the general recurrence relation Eqn. (4.14) at node n-1 the deflection at node (n-1) can be obtained as,

$$W2(n-1) = \frac{C(n-1)}{1+B(n-1)}$$

Knowing W2(n), W2(n-1) and all other recurrence coefficients, by the process of back substitution, the deflections at

other nodes can be evaluated.

4.4.3 Pin-Pin End Conditions:

The initial shape satisfying natural and geometric boundary conditions corresponding to pinned-pinned end conditions is given by,

$$w_1 = \frac{C}{T}\left[1.5\frac{z(I)}{z_m} - 7.0\frac{z(I)^3}{z_m^3} + 11.0\frac{z(I)^4}{z_m^4} - 4.5\frac{z(I)^5}{z_m^5}\right] \tag{3.31}$$

Differentiating Eqn. (3.31) and substituting in Eqn. (4.7) and (4.8) gives,

$$\begin{aligned} RHS'(I) = & -\alpha\,(1-\beta_1 z(I))\frac{C}{T}\left[-42\frac{z(I)}{z_m^3} + 132\frac{z(I)^2}{z_m^4}\right.\\ & \left. - 90\frac{z(I)^3}{z_m^5}\right] + \alpha\beta_1\frac{C}{T}\left[\frac{1.5}{z_m} - 21\frac{z(I)^2}{z_m^3}\right.\\ & \left. + 44\frac{z(I)^3}{z_m^4} - 22.5\frac{z(I)^4}{z_m^5}\right] \quad \text{for } 0 \le i \le n_1 \end{aligned} \tag{4.20}$$

$$\begin{aligned} RHS'(I) = & -\alpha(1-\beta_1 z_1 + \beta_2 z_1 - \beta_2 z(I))\frac{C}{T}\left[-42\frac{z(I)}{z_m^3}\right.\\ & \left. + 132\frac{z(I)^2}{z_m^4} - 90\frac{z(I)^3}{z_m^5}\right] + \alpha\beta_2\frac{C}{T}\left[\frac{1.5}{z_m}\right.\\ & \left. - 21\frac{z(I)^2}{z_m^3} + 44\frac{z(I)^3}{z_m^4} - 22.5\frac{z(I)^4}{z_m^5}\right]\\ & \text{for } n_1 \le i \le n \end{aligned} \tag{4.21}$$

Writing the top boundary conditions in finite difference form,

$$W2(0) = 0, \qquad \left(\frac{d^2w_2}{d_z^2}\right)_{z=0} = \frac{W2(1) - 2W2(0) + W2(-1)}{2h} = 0$$

gives $W2(1) = -W2(-1)$.

Writing Eqn. (4.11) at node 1, and substituting $W2(0) = 0$ and $W2(1) = -W2(-1)$ the coefficients $A(1)$, $B(1)$ and $C(1)$ can be obtained as,

$$A(1) = \frac{-FA(1)}{-2.0 + FB(1)}, \quad B(1) = \frac{-2.0}{-2.0 + FB(1)},$$

$$C(1) = \frac{RHS(1)}{-2.0 + FB(1)}$$

Writing Eqn. (4.11) at node 2 and substituting $W2(0)$, and $W2(1)$ the coefficients $A(2)$, $B(2)$ and $C(2)$ can be obtained as,

$$A(2) = -\frac{[FA(2) + B(1)\ FC(2)]}{FB(2) + A(1)\ FC(2)},$$

$$B(2) = \frac{-2.0}{FB(2) + A(1)\ FC(2)}$$

$$C(2) = \frac{RHS(2) - C(1)\ FC(2)}{FB(2) + A(1)\ FC(2)}$$

The recurrence coefficients at any general node i are given by Eqn. (4.15), (4.16) and (4.17).

Writing the bottom boundary conditions in finite difference form,

$$W2(n) = 0, \quad \left(\frac{d^2 w_2}{d_z^2}\right)_{z=z_m} = \frac{W2(n+1) - 2W2(n) + W2(n-1)}{h^2} = 0$$

gives, $W2\,(n+1) = -W2\,(n-1)$

Writing the general recurrence relation (4.14) at node (n-1) and substituting bottom boundary conditions the deflection at (n-1)th node can be obtained as,

$$W2(n-1) = \frac{C(n-1)}{1 + B(n-1)}$$

Knowing W2(n), W2(n-1) and all the recurrence coefficients, by the process of back substitution the deflections at all the nodes can be evaluated.

4.5 Analysis of Bent Friction Piles in Cohesionless Soil:

The assumption of soil modulus constant with depth in each layer may not be necessarily true in all cases. For normally consolidated clays and especially for granular soils the assumption of soil modulus varying linearly with depth is more appropriate. When the soil modulus varies linearly it is reasonable to assume the variation of axial load in the pile to be parabolic. For this assumption Eqn. (4.2) reduces to,

$$EI \frac{d^4 u_{2t}}{dy^4} + P\left(1 - \varphi_1 \frac{y^2}{L^2}\right) \frac{d^2 u_{2t}}{dy^2} - 2P\varphi_1 \frac{y}{L^2} \frac{du_{2t}}{dy} + n_{ht}\, y\, u_{2t}$$

$$= -P\left(1 - \varphi_1 \frac{y^2}{L^2}\right) \frac{d^2 u_1}{dy^2} + 2P\varphi_1 \frac{y}{L^2} \frac{du_1}{dy}$$

$$\text{for } 0 \leq y \leq L_1 \qquad (4.22)$$

$$EI \frac{d^4u_{2b}}{dy^4} + P(1-\varphi_1 \frac{y_1^2}{L^2} - \varphi_2 \frac{y^2+y_1^2-2yy_1}{L^2}) \frac{d^2u_{2b}}{dy^2}$$

$$- 2P\varphi_2 (\frac{y}{L^2} - \frac{y_1}{L^2}) \frac{du_{2b}}{dy} + n_{hb}\, y\, u_{2b}$$

$$= -P(1-\varphi_1 \frac{y_1^2}{L^2} - \frac{\varphi_2 y_1^2}{L^2} - \varphi_2 \frac{y^2-2yy_1}{L^2}) \frac{d^2u_1}{dy^2}$$

$$+ 2P\varphi_2 (\frac{y}{L^2} - \frac{y_1}{L^2}) \frac{du_1}{dy} \quad \text{for } L_1 \leq y \leq L \tag{4.23}$$

where φ_1 and φ_2 are skin friction factors for cohesionless soils for the top layer and bottom layer respectively.

n_{ht}, n_{hb} are the coefficients of modulus variation for the top and bottom layer respectively.

The remaining terms are explained previously.

The relative stiffness factor T is defined as $T = [\frac{EI}{n_{ht}}]^{1/5}$

Non-dimensionalizing the parameters with T as shown in previous sections, the following relations can be written,

$$z = \frac{y}{T}, \quad z_m = \frac{L}{T}, \quad z_1 = \frac{L_1}{T}, \quad w_1 = \frac{u_1}{T}, \quad w_2 = \frac{u_2}{T}, \quad w = \frac{u}{T}$$

Substituting the above non-dimensional parameters in the Eqns. (4.22) and (4.23) they can be written in non-dimensional form as,

$$\frac{d^4w_{2t}}{dz^4} + \alpha\,(1-\beta_1 z^2) \frac{d^2w_{2t}}{dz^2} - 2\alpha_1\beta_1 z \frac{dw_{2t}}{dz} + w_{2t}\, z$$

$$= -\alpha\,(1-\beta_1 z^2) \frac{d^2w_1}{dz^2} + 2\alpha_1\beta_1 z \frac{dw_1}{dz} \quad \text{for } 0 \leq z \leq z_1 \tag{4.24}$$

$$\frac{d^4 w_{2b}}{dz^4} + \alpha(1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z^2-2zz_1))\ \frac{d^2 w_{2b}}{dz^2}$$

$$+ \alpha\ (2\beta_2 z_1 - 2\beta_2 z)\ \frac{dw_{2b}}{dz} + R_o\ w_{2b} z$$

$$= -\alpha(1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z^2-2zz_1))\ \frac{d^2 w_1}{dz^2}$$

$$+ 2\alpha\beta_2\ (z-z_1)\ \frac{dw_1}{dz} \quad \text{for } z_1 \leq z \leq z_m \qquad (4.25)$$

where,

$$\beta_1 = \frac{\varphi_1}{z_m{}^2} \quad \text{and} \quad \beta_2 = \frac{\varphi_2}{z_m{}^2}, \qquad R_o = \frac{n_{hb}}{n_{ht}}$$

The remaining terms are explained previously.

At any general node i the right hand side of the above differential equation can be written as,

$$RHS'(I) = -\alpha(1-\beta_1 z(I)^2)\ \left(\frac{d^2 w_1}{dz^2}\right)_{z=z(I)}$$

$$+ 2\alpha\beta_1 z(I)\ \left(\frac{dw_1}{dz}\right)_{z=z(I)} \quad \text{for } 0 \leq z \leq z_1 \qquad (4.26)$$

$$RHS'(I) = -\alpha(1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z^2-2z(I)\ z_1))\left(\frac{d^2 w_1}{dz^2}\right)_{z=z(I)}$$

$$+ 2\alpha\beta_2\ (z(I) - z_1)\ \left(\frac{dw_1}{dz}\right)_{z=z(I)}$$

$$\text{for } z_1 \leq z \leq z_m \qquad (4.27)$$

Substituting the difference equations (4.1) in Eqns. (4.24) and (4.25) the left hand side of the equation at any node i can be written as,

$$\frac{1}{2h^4}\,[2W2\,(I+2) + FA(I)\,W2(I+1) + FB(I)\,W2(I)$$

$$+ FC(I)\,W2(I-1) + 2W2\,(I-2)] \qquad (4.28)$$

where,

$$FA(I) = -8.0 + 2\alpha h^2\,(1-\beta_1 z(I)^2) - 2\alpha\beta_1 h^3 z(I)$$

$$FB(I) = 12.0 - 4\alpha h^2\,(1-\beta_1 z(I)^2) + 2h^4\,z(I)$$

$$FC(I) = -8.0 + 2\alpha h^2\,(1-\beta_1 z(I)^2) + 2\alpha\beta_1 h^3\,z(I)$$

$$\text{for } 0 \leq i \leq n_1-5$$

$$FA(I) = -8.0 + 2\alpha h^2\,(1-\beta_1 z(I)^2) - 2\alpha\beta_1 h^3 z(I)$$

$$FB(I) = 12.0 - 4\alpha h^2\,(1-\beta_1 z(I)^2) + 2h^4\,[z(n_1-5)$$

$$+ \frac{R_o\; z(n_1+5) - z(n_1-5)}{10}\,(i+5-n_1)]$$

$$FC(I) = -8.0 + 2\alpha h^2\,(1-\beta_1 z(I)^2) + 2\alpha\beta_1 h^3 z(I)$$

$$\text{for } n_1-5 \leq i \leq n_1$$

$$FA(I) = -8.0 + 2\alpha h^2\,(1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2\,(z(I)^2-2z(I)z_1))$$

$$- 2\alpha\beta_2 h^3(z(I) - z_1)$$

$$FB(I) = 12.0 - 4\alpha h^2\,(1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z(I)^2-2z(I)z_1))$$

$$+ 2h^4\,[\,z(n_1-5) + \frac{R_o\; z(n_1+5) - z(n_1-5)}{10}\,(i+5-n_1)$$

$$FC(I) = -8.0 + 2\alpha h^2\,(1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z(I)^2-2z(I)z_1))$$

$$+ 2\alpha\beta_2 h^3(z(I) - z_1) \qquad \text{for } n_1 \leq i \leq n_1+5$$

$$FA(I) = -8.0 + 2\alpha h^2 \, (1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z(I)^2-2z(I)z_1)) - 2\alpha\beta_2 h^3 \, (z(I)-z_1)$$

$$FB(I) = 12.0 - 4\alpha h^2 \, (1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z(I)^2-2z(I)z_1)) + 2h^4 \, R_o \, z(I)$$

$$FC(I) = -8.0 + 2\alpha h^2 \, (1-\beta_1 z_1{}^2 - \beta_2 z_1{}^2 - \beta_2(z(I)^2-2z(I)z_1)) + 2\alpha\beta_2 \, h^3 \, (z(I)-z_1)$$

for $n_1+5 \leq i \leq n$

Defining RHS (I) = $2h^4$ RHS$'$(I)

the governing differential equations (4.24) and (4.25) can be written at any node i in terms of additional deflections of neighbouring nodes as,

$$2W2(I+2) + FA(I)\,W2(I+1) + FB(I)\,W2(I) + FC(I)\,W2(I-1) + 2W2(I-2) = RHS\,(I) \qquad (4.29)$$

The differential Eqns. (4.24) and (4.25) can be solved to find the additional deflections at all nodes for fix-fix, fix-pin and pin-pin end conditions exactly on similar lines of cohesive soil. The only difference between these two cases is the values of RHS (I), FA(I), FB(I), FC(I) differ one another.

4.6 Analysis of Bent Friction Piles in Non-Homogeneous Soils:

In this section the effect of uniformly varying soil with the polynomial variation of soil modulus and parabolic variation of axial load in the pile is studied. The governing differential equation for this case takes the form:

$$EI \frac{d^4 u_2}{dy^4} + \frac{d}{dy} \left[P(1- \varphi \frac{y^2}{L^2}) \frac{du}{dy} \right] + (K_o + K_1 y^n) u_2 = 0 \tag{4.30}$$

The relative stiffness factor T is defined as,

$$T = \left[\frac{EI}{K_1} \right]^{1/(n+4)}$$

Non-dimensionalizing the parameters with T as shown in previous sections, the following relations can be written.

$$z = \frac{y}{T}, \quad z_m = \frac{L}{T}, \quad w_1 = \frac{u_1}{T}, \quad w_2 = \frac{u_2}{T}, \quad w = \frac{u}{T}$$

Substituting the above non-dimensional parameters in Eqn.(4.30), it can be written in non-dimensional form as,

$$\frac{d^4 w_2}{dz^4} + \alpha(1-\beta z^2) \frac{d^2 w_2}{dz^2} - 2\alpha\beta z \frac{dw_2}{dz} + (r_o + z^n) w_2$$
$$= -\alpha (1-\beta z^2) \frac{d^2 w_1}{dz^2} + 2\alpha\beta z \frac{dw_1}{dz} \tag{4.31}$$

where,

$$\beta = \frac{\varphi}{z_m^2} \quad \text{and} \quad r_o = \frac{K_o T^4}{EI}$$

The remaining terms are explained previously.

4.6.1 Solution:

At any general node i, the right hand side of the differential equation (4.31) can be written as,

$$RHS'(I) = -\alpha\ (1-\beta z(I)^2)\ \left(\frac{d^2w_1}{dz}\right)_{z=z(I)} + 2\alpha\beta z(I)\ \left(\frac{dw_1}{dz}\right)_{z=z(I)}$$

Substituting difference equations (4.1) in equation (4.31) the left hand side of the differential equation about node i in finite difference form can be expressed as,

$$\frac{1}{2h^4}\ [2W2\ (I+2) + FA(I)\ W2(I+1) + FB(I)\ W2(I) + FC(I)\ W2(I-1) + 2W2\ (I-2)]$$

where,

$$FA(I) = -8.0 + 2\alpha h^2\ (1-\beta z(I)^2) - 2\alpha\beta z(I)\ h^3$$

$$FB(I) = 12.0 - 4\alpha h^2\ (1-\beta z(I)^2) + 2h^4\ (r_o + z^n)$$

$$FC(I) = -8.0 + 2\alpha h^2\ (1-\beta z(I)^2) + 2\alpha\beta h^3 z(I)$$

Defining $RHS\ (I) = 2h^4\ RHS'(I)$,

the governing differential equation at any node i can be expressed in terms of additional deflections of neighbouring points as,

$$2W2(I+2) + FA(I)\ W2(I+1) + FB(I)\ W2(I) + FC(I)\ W2(I-1) + 2W2(I-2) = RHS\ (I)$$

The above equation can be solved for Fix-Fix, Fix-Pin, and Pin-Pin end conditions exactly on similar lines of cohesive soils with friction.

4.7 Load Carrying Capacity of Bent Friction Piles:

Finite difference technique used to solve the differential equations for various cases mentioned in the previous sections provide the additional deflections under axial loading. The procedure to obtain the load carrying capacity from additional deflections is as follows.

The calculated additional deflections are added to the initial deflections to get the total deflections, $w = w_1 + w_2$. From the finite difference expression for bending moment at node i,

$$\left(\frac{d^2w}{dz^2} \right) = \frac{w_{(i+1)} - 2w_{(i)} + w_{(i-1)}}{h^2}$$

where, w_i represents the total deflection at node i the bending moments at all nodes are calculated.

The criteria for design is,

$$\frac{f_a}{F_a} + \frac{C_m\ f_b}{(1 + \frac{f_a}{F_e'})\ F_b} \leq 1 \qquad (3.17)$$

The various terms in the above equations are explained in Sec. 3.2. For friction piles in layered cohesive soil,

$$f_a = \frac{P}{A}(1-\beta_1 z) \quad \text{for} \quad 0 \leq z \leq z_1$$

$$= \frac{P}{A}(1-\beta_1 z_1 - \beta_2(z-z_1)) \quad \text{for} \quad z_1 \leq z \leq z_m$$

where,

$$\beta_1 = \frac{\varphi_1}{z_m} \qquad \text{and} \qquad \beta_2 = \frac{\varphi_2}{z_m}$$

For friction piles in layered cohesionless soil,

$$f_a = \frac{P}{A}(1-\beta_1 z^2) \qquad \text{for } 0 \leq z \leq z_1$$

$$= \frac{P}{A}(1-\beta_1 z_1^{\,2} - \beta_2 (z-z_1)^2) \text{ for } z_1 \leq z \leq z_m$$

where,

$$\beta_1 = \frac{\varphi_1}{z_m^{\,2}}, \qquad \beta_2 = \frac{\varphi_2}{z_m^{\,2}}$$

For friction piles in non-homogeneous soil,

$$f_a = \frac{P}{A}(1 - \beta z^2)$$

where, $\beta = \dfrac{\varphi}{z_m^{\,2}}$

The highest axial load which causes the axial and bending stresses along the length of bent pile satisfying equation (3.17) is determined by iteration process and is expressed as percentage capacity of straight end bearing pile.

4.8 Analysis of Bent Piles Subjected to Down Drag:

When a pile is driven through compressible strata to a relatively firm material, there is a tendency for the compressible strata to consolidate by an amount in excess of the displacement of the pile under applied load. In such a case the soil around the pile tries to hang up on the pile, exerting a downward drag. The drag may be of considerable magnitude and consequently the load transferred to the bottom portion of

bent pile is increased. The analysis of bent piles with the drag force on the entire length of the pile was studied by Kurma Rao (1975) and Arumugam (1977). In this investigation the drag force factor is varied in both top layer and bottom layer. The solution is same as that of friction piles with the exception that the skin friction factor acquires negative sign.

4.9 Results and Discussion:

To check the validity of the finite-difference method a comparison is made between closed form solution and finite-difference solution with respect to load carrying capacity. Table 4.1 shows the load carrying capacities of a smooth bent pile with fix-fix end conditions in a homogeneous soil for different initial pile offsets. The parameters $z_m = 5.0$, $R_o = 1.0$, $T/r = 22.0$ are kept constant and C/T is varied between 0.01 and 0.05. Table 4.2 shows the load carrying capacities of a smooth bent fix-fix pile in a layered soil. The parameters $z_m = 5.0$, $z_1 = 2.5$, $C/T = 0.01$ and $T/r = 22.0$ are kept constant and R_o is varied between 0.25 and 4.0. As seen from these tables the finite-difference method gives consistently slightly higher values, however, this difference is insignificant as the maximum error is 6 percent in the worst case.

To study the effect of friction on the variation of additional deflections and bending moments the parameters

$z_m = 5.0$, $R_o = 1.0$, $C/T = 0.01$ and $T/r = 22.0$ are kept constant. The friction factor is varied between -1.0 to +1.0. Fix-fix end conditions are considered for the pile. In cohesive soils, the effect of friction on the variation of additional deflections and additional bending moments is shown in Fig. 4.4a and 4.4b respectively. In cohesionless soils the effect of friction on the variation of additional deflections and additional bending moments is shown in Fig. 4.5a and 4.5b respectively. For both types of soil it is seen that in general as the friction increases the additional deflections and the additional bending moments decreases. But the amount of reduction in deflection for cohesionless soils is about 50 percent of that in cohesive soils. In both cases the reduction is more in the bottom part of the pile than in the upper one. Effect of negative skin friction ($\varphi = -1$) is more severe in both cases.

To study the effect of friction on the percentage load carrying capacity of an initially bent pile having different lengths in cohesive and cohesionless soils, the parameters $C/T = 0.01$, $R_o = 1.0$, $T/r = 22.0$ are kept constant, z_m is varied between 2 to 6 and the friction factor is varied between -1.0 to 1.0. Figs. 4.6a and 4.6b show the variation of percentage load carrying capacity with z_m for fix-fix and fix-pin end conditions respectively for cohesive soils. Figs. 4.7a and 4.7b show similar graphs for

cohesionless soil. It is seen from these figures that as the friction increases the percentage load carrying capacity increases and for negative skin friction the load carrying capacity decreases. In all the cases, for the same friction factor, the increase in the load carrying capacity is higher for longer piles. The effect of negative skin friction is more severe in fix-fix pile than in fix-pin pile. For pin-pin pile with the shape under consideration the load carrying capacity is 100 percent of the corresponding end bearing straight piles i.e. there is no change in load carrying capacity due to initial bending. This is because the maximum bending stress occur at a depth of 70 to 80 percent of pile length and at that depth the addition of bending and axial stresses are less than the allowable stress for a load on the pile head corresponding to 100 percent capacity of end bearing pile.

To study the effect of friction on the percentage load carrying capacity of an initially bent pile having different initial pile offsets in cohesive and cohesionless soils the parameters $z_m = 5.0$, $R_o = 1.0$, $T/r = 22.0$ are kept constant, C/T is varied between 0.01 to 0.05 and friction factor is varied between -1.0 and +1.0. Figs. 4.8a and 4.8b show the effect of friction on the percentage load carrying capacity of fix-fix and fix-pin piles respectively for cohesive soils. Figs. 4.9a and 4.9b show similar graphs for cohesionless soils.

It is observed from these figures that friction significantly increases the load carrying capacity at smaller pile offsets. As the offset value increases the effect of friction decreases and approximately at C/T = 0.04, it becomes negligibly small.

To study the effect of friction on the capacity of bent piles in cohesive soils at different T/r values, the parameters $z_m = 5.0$, $R_o = 1.0$, C/T = 0.01 are kept constant and T/r is varied between 10 and 26. Figs. 4.10a and 4.10b show the effect of slenderness parameter on the capacity of bent piles for different friction factors for fix-fix and fix-pin end conditions respectively. It can be seen from these figures that the load carrying capacity attained certain maximum value between T/r = 15 and 20 and then gradually reduces. This is because, for highly slender members, for a given offset even though the initial bending stresses are small the additional bending stresses due to imposed load are large making the capacity of bent piles small. For bulky sections the initial bending stresses for the given offset are high thus reducing the capacity of bent piles. For cohesionless soil since practically there is no increase in the capacity with friction it is not shown. On the same reasons as mentioned above pin-pin end conditions are not considered.

In order to study the effect of layering (R_o) on the capacity of bent friction piles the friction factor for the top layer is kept constant ($\varphi_1 = 0.5$) and the friction factor for the bottom layer (φ_2) is varied as shown in Table 4.3.

Figs. 4.11a and 4.11b show the effect of R_o on the load carrying capacities of fix-fix and fix-pin piles respectively. The parameters $z_m = 5.0$, $z_1 = 2.5$, $C/T = 0.01$ and $T/r = 22.0$ are kept constant and R_o is varied from 0.25 to 4.0. In both figures, it is seen that as R_o increases i.e. when the strength of bottom layer increases the load carrying capacity also increases. However, the percentage increase is very small (less than 0.2 percent) and for all practical purposes it can be neglected. This may be explained from the fact that the increase in load carrying capacity due to friction is compensated by the reduced resistance of the soil because of the reduced additional deflection. Figs. 4.12a and 4.12b show similar graphs for cohesionless soil for fix-fix and fix-pin end conditions respectively.

Figs. 4.11c and 4.11d show the effect of thickness of top layer (z_1) on the load carrying capacities of fix-fix and fix-pin piles respectively when $R_o = 0.25$ (i.e. when the bottom layer is weaker than the top layer) and $R_o = 4.0$ (i.e. when the bottom layer is stronger than the top layer). Similar graphs are drawn in Figs. 4.12c and 4.12d for

cohesionless soil. It is seen from all these figures that when the bottom layer is weaker than top layer increase in depth of top layer increases the capacity of bent pile and vice-versa.

Figs. 4.11e and 4.11f show the effect of negative skin friction of the upper layer for fix-fix and fix-pin end conditions respectively. The parameters $z_m = 5.0$, $z_1 = 2.5$, $R_o = 4.0$, $C/T \doteq 0.01$, $T/r = 22.0$ and $\varphi_2 = 0.5$ are kept constant and φ_1 is varied. Similar graphs are drawn for parabolic variation of negative skin friction in Figs. 4.12e and 4.12f. In all the cases, it is seen that as the negative skin friction factor increases the load carrying capacity reduces. While there is a reduction of 5 percent in the load carrying capacity in case of soils having linear variation of negative skin friction there is only 1 percent reduction in the case of soils having parabolic variation of negative skin friction.

The effect of polynomial variation of subgrade modulus on bent friction piles is shown in Fig. 4.13. For fix-fix and fix-pin end conditions friction factor is taken as 0.5 and for pin-pin end conditions friction factor is taken as 0.0 for the reasons mentioned previously. Figs. 4.13a , 4.13b and 4.13c show the variation of capacity of bent pile for $r_o = 0.25$ and varying n for fix-fix, fix-pin, pin-pin end

conditions respectively. Figs. 4.13d, 4.13e and 4.13f show the variation of capacity for $n = 0.75$ and for varying r_o values. The effect of increase in either n or r_o is to increase the capacity of bent pile. However, for practical purposes the increase can be neglected.

Table 4.1: Comparison of closed form and finite difference solution in homogeneous soil.

$\frac{C}{T}$	Capacity in percentage		Percentage error
	Closed Form	Finite Difference	
0.01	71.57	71.75	0.28
0.02	52.30	52.61	0.59
0.03	37.04	37.50	1.24
0.04	23.75	24.34	2.48
0.05	12.08	12.82	6.12

Table 4.2: Comparison of closed form and finite difference solution in layered cohesive soil.

R_o	Capacity in percentage		Percentage error
	Closed form	Finite Difference	
0.25	71.52	71.69	0.24
0.50	71.54	71.71	0.24
0.75	71.56	71.73	0.24
1.00	71.57	71.75	0.25
2.00	71.61	71.79	0.25
4.00	71.64	71.82	0.25

Table 4.3: Variation of φ_2 with R_o.

R_o	0.25	0.50	0.75	1.0	2.0	4.0
2	0.2	0.3	0.4	0.5	0.7	0.9

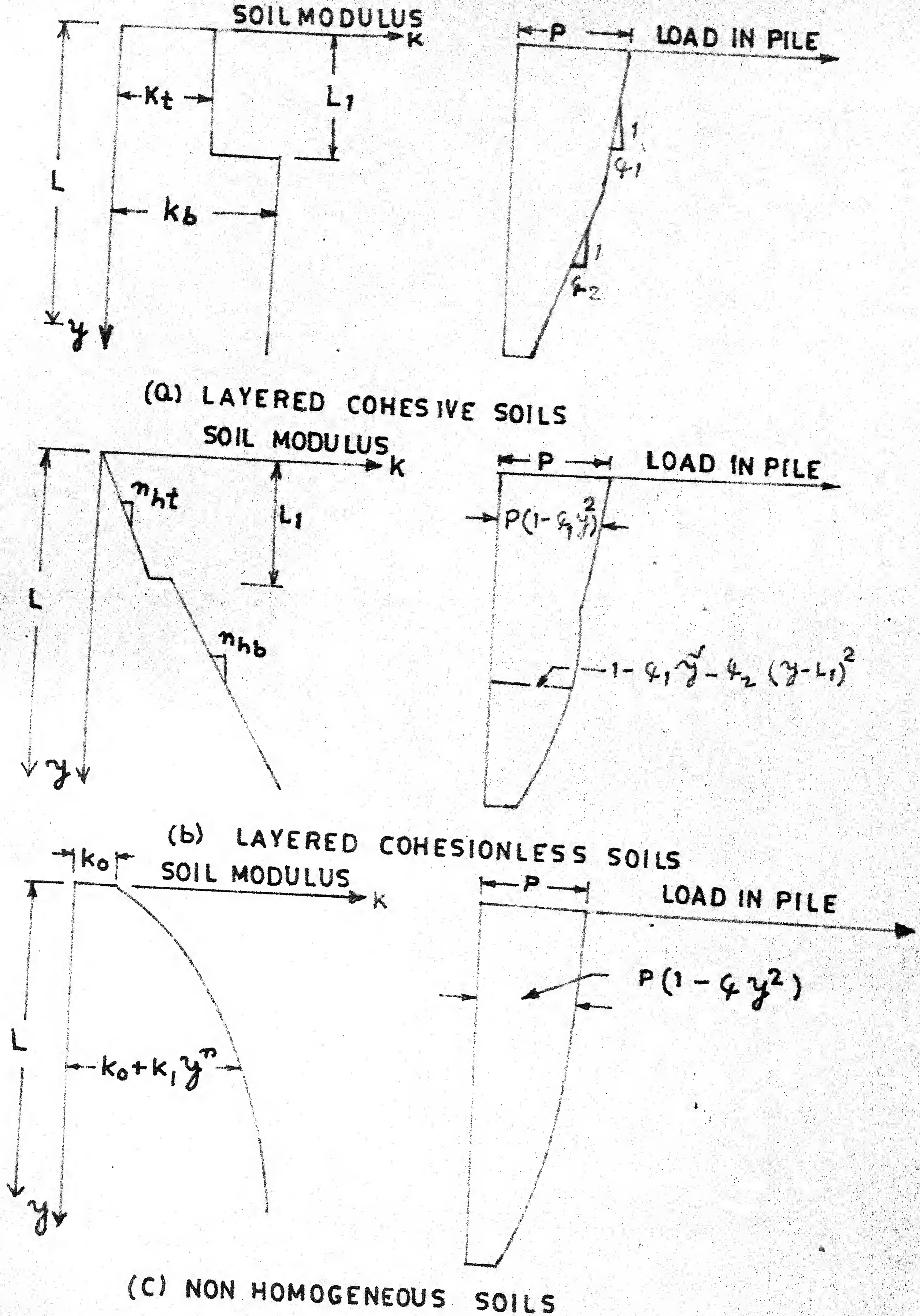


FIG. 4.1 VARIATION OF SOIL MODULUS AND AXIAL LOAD IN PILE WITH DEPTH

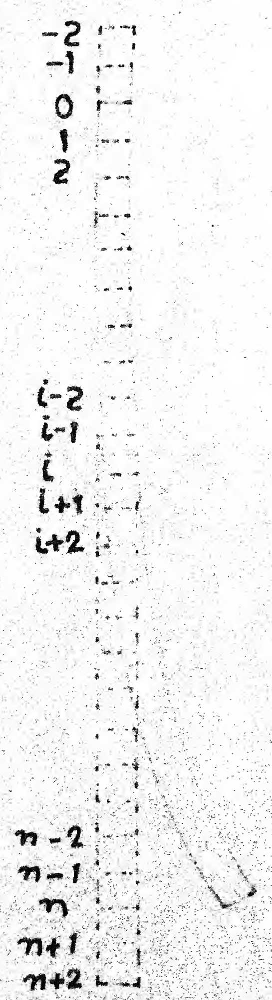


FIG.4·2 BENT PILE DIVIDED INTO SEGMENTS

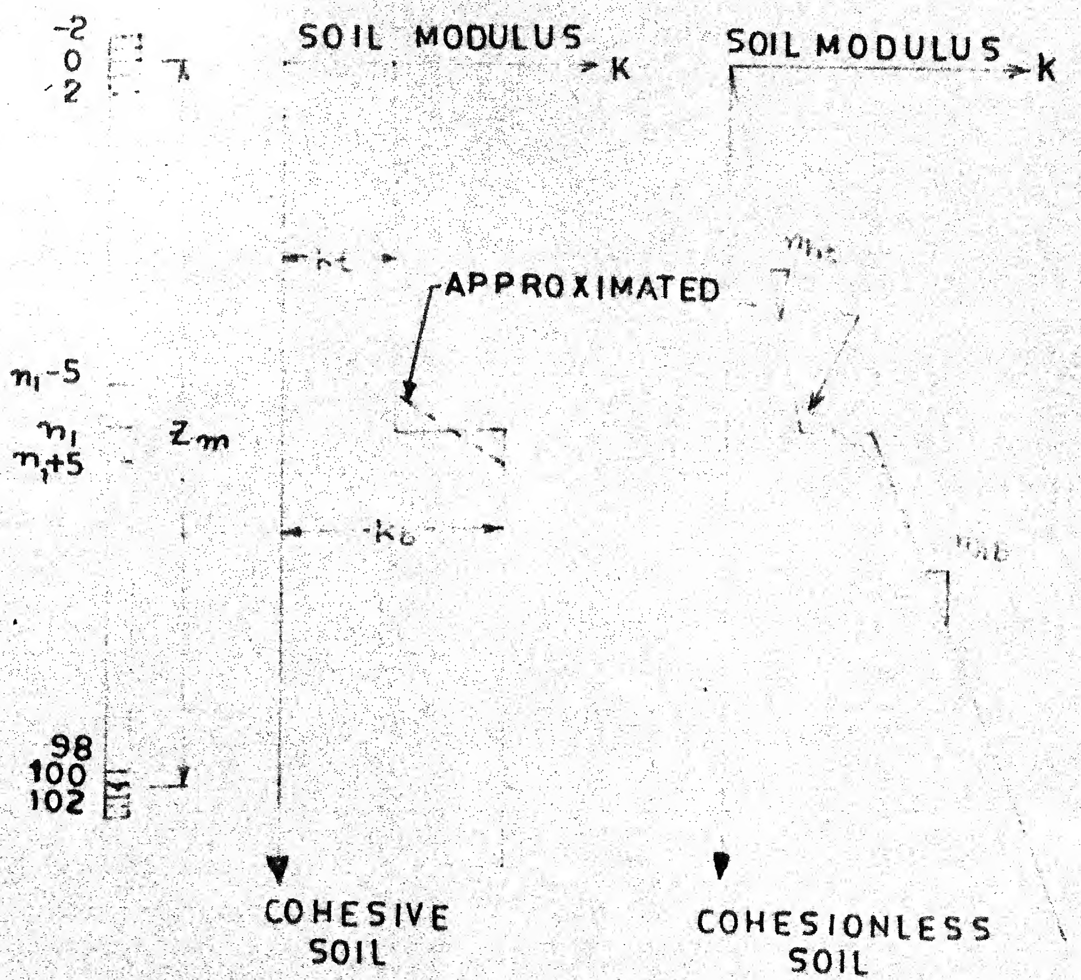


FIG.4·3 APPROXIMATED VARIATION OF SOILMODULUS

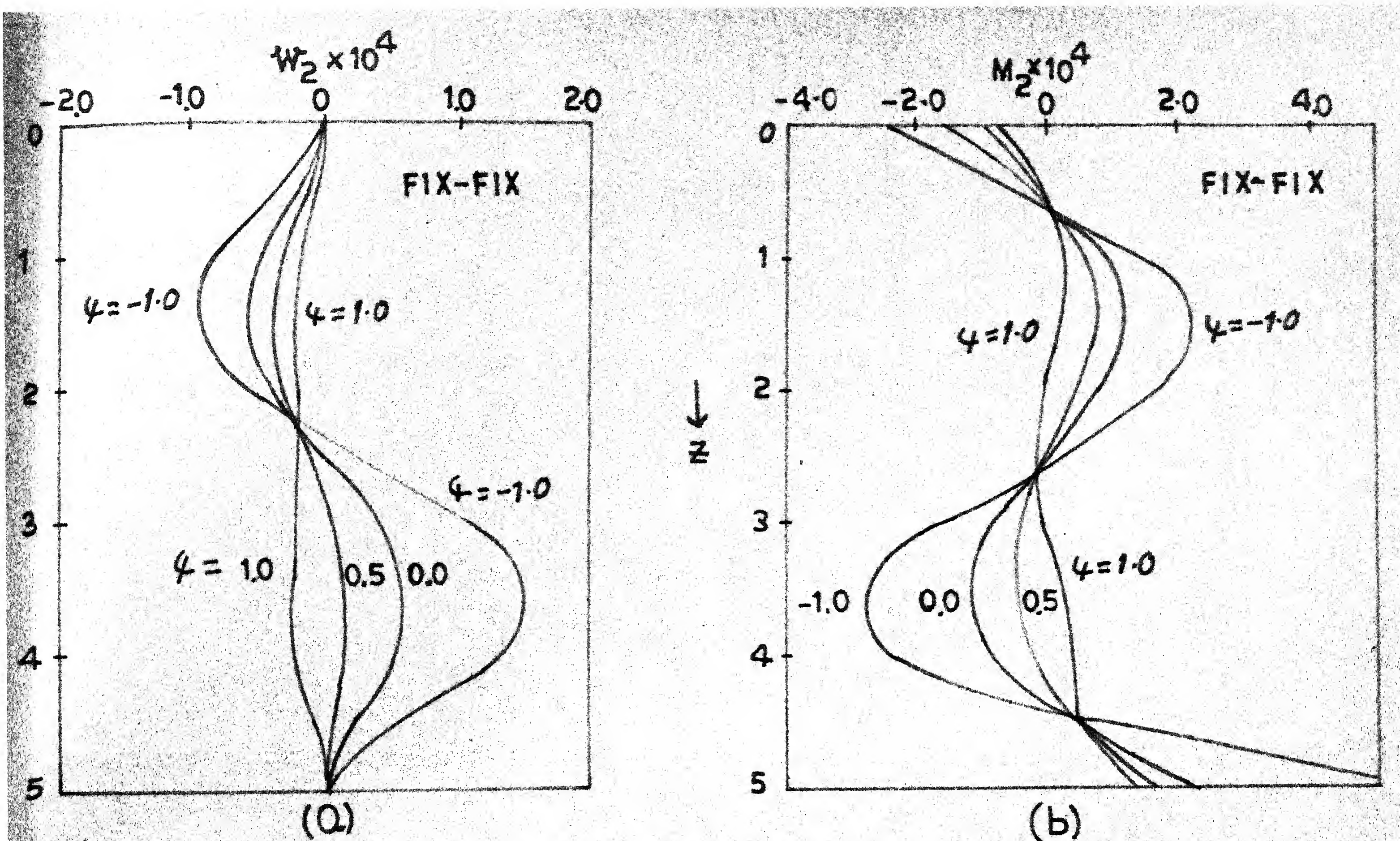
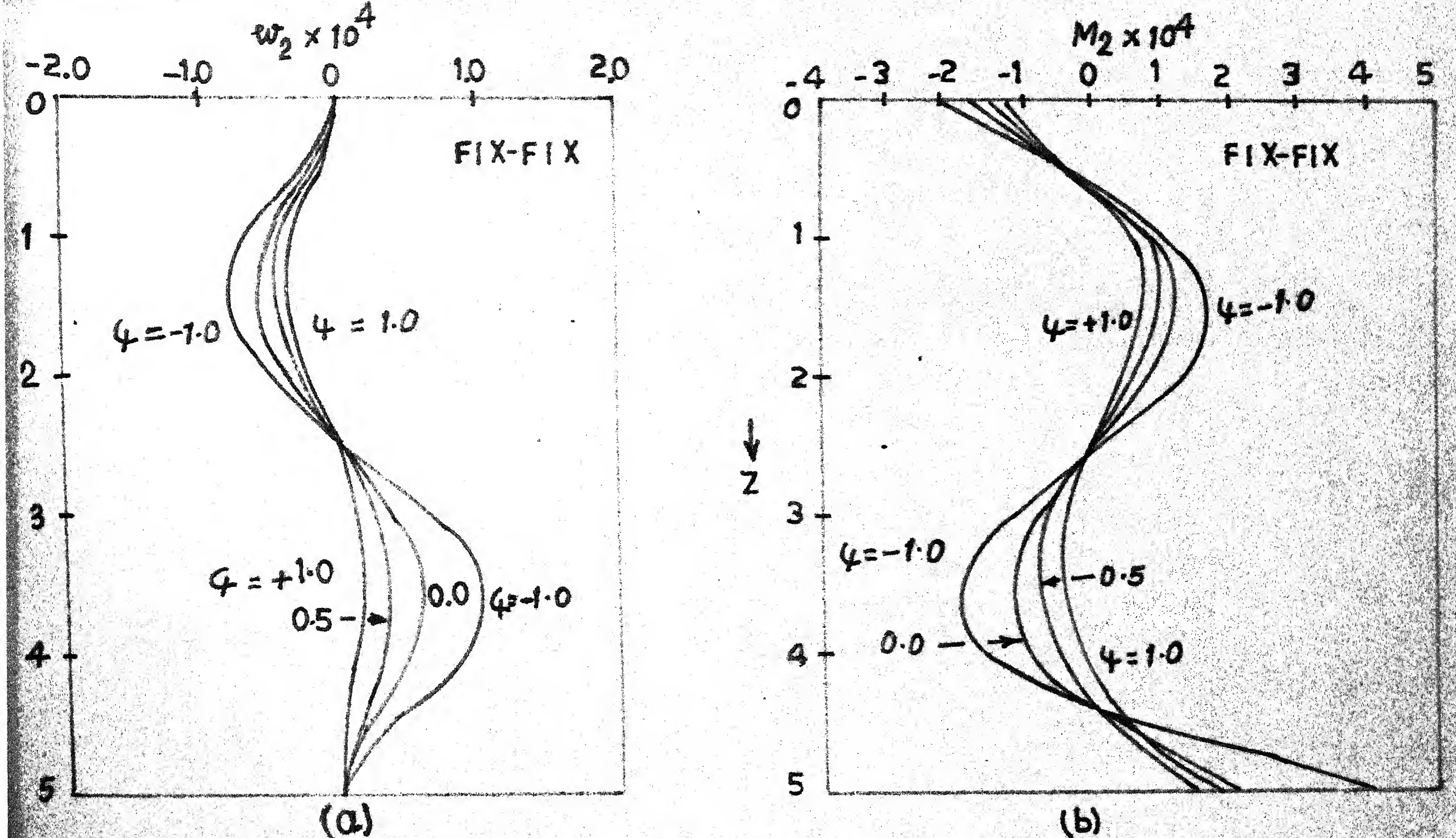
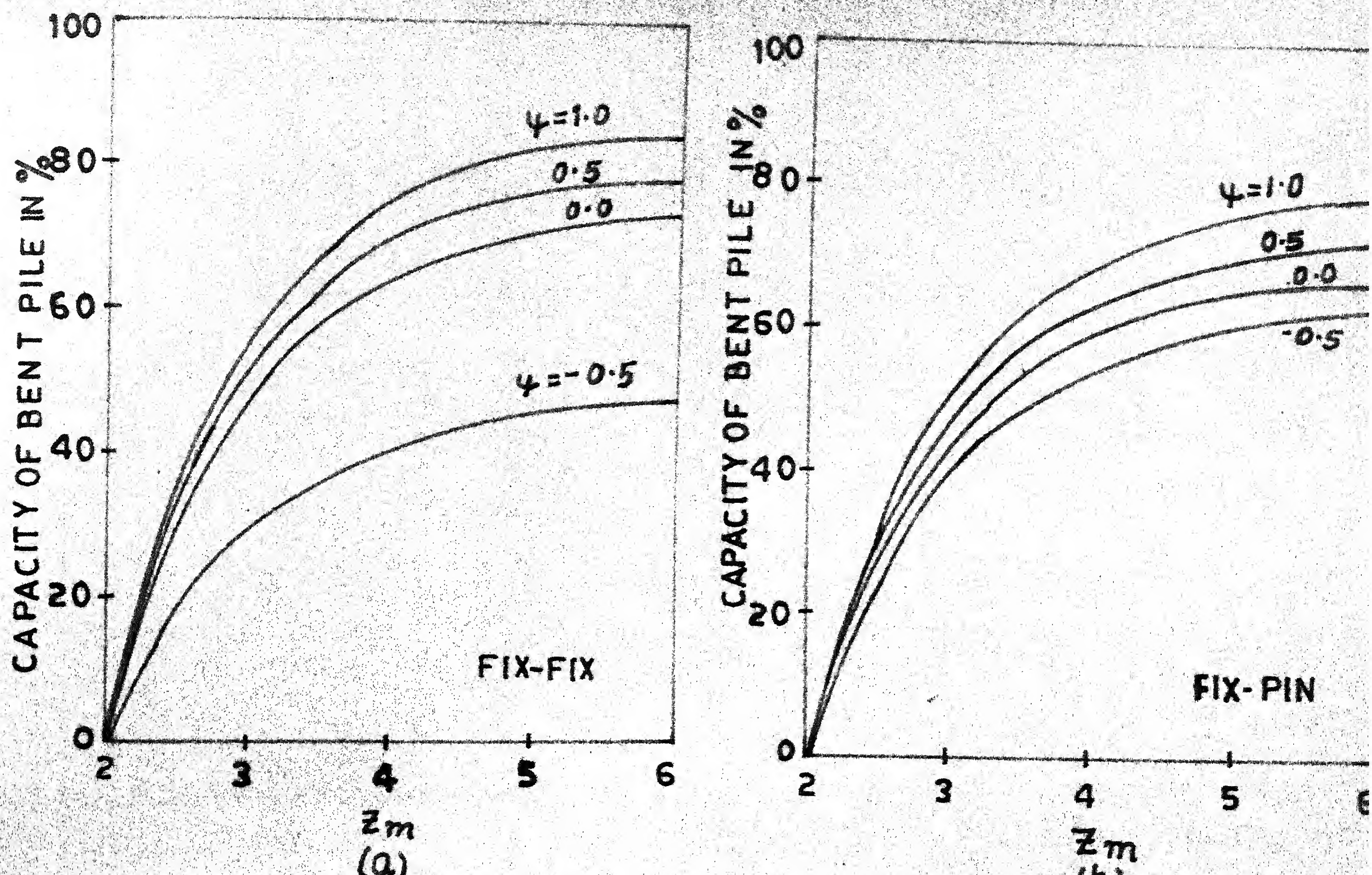


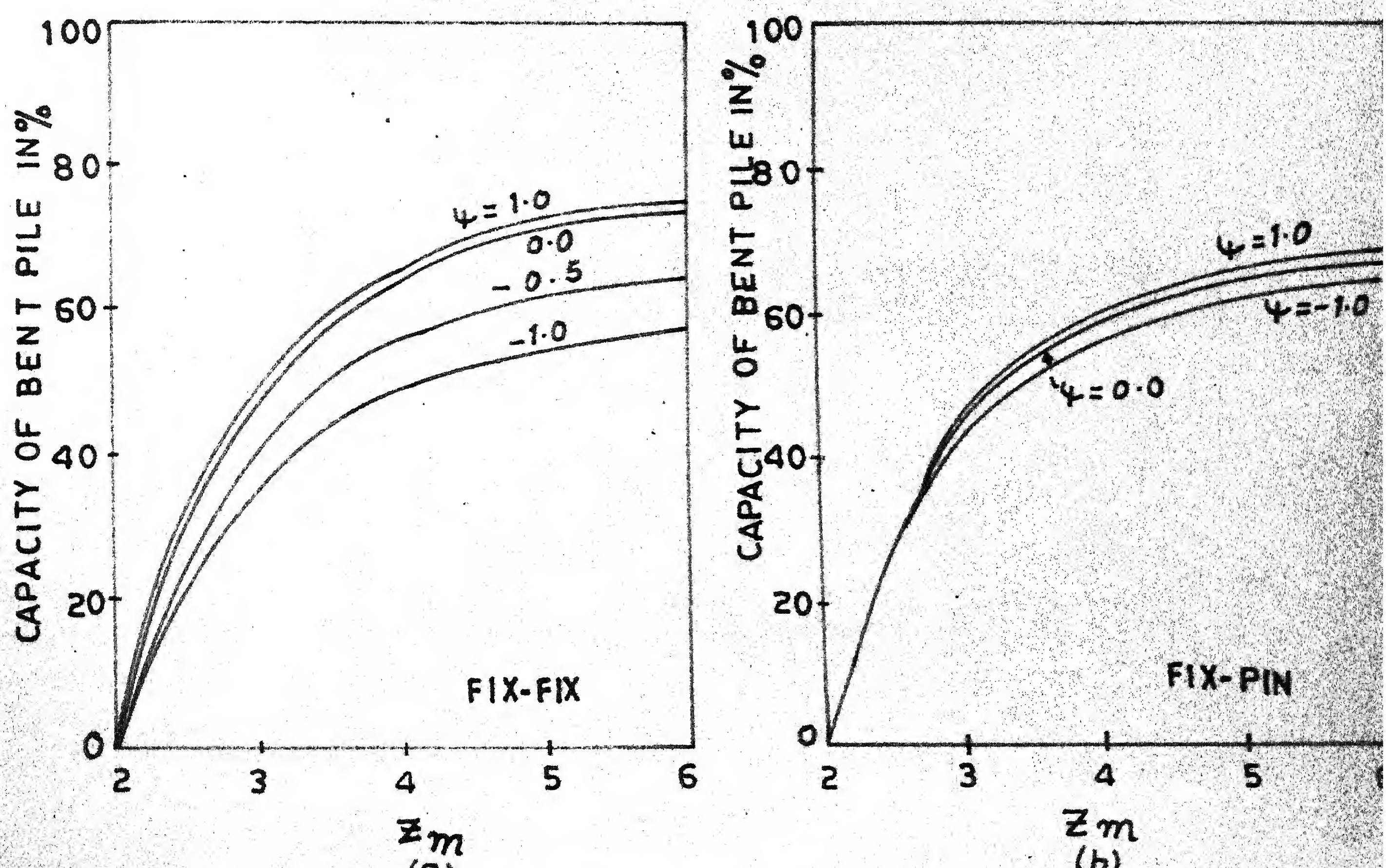
FIG.4.4 EFFECT OF FRICTION ON w_2 AND M_2 IN COHESIVE SOILS



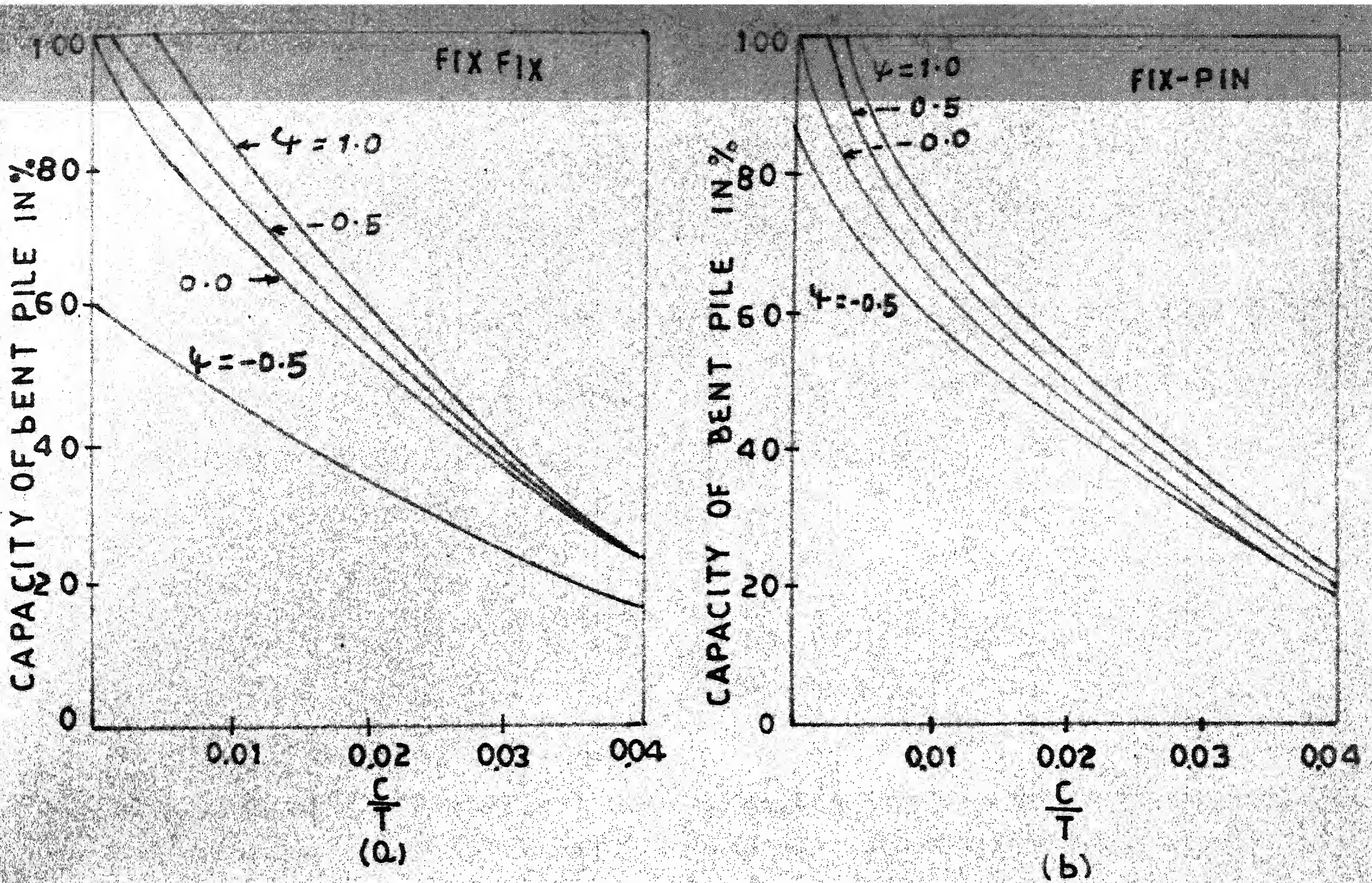
IG.4.5 EFFECT OF FRICTION ON w_2 AND M_2 IN COHESIONLESS SOILS



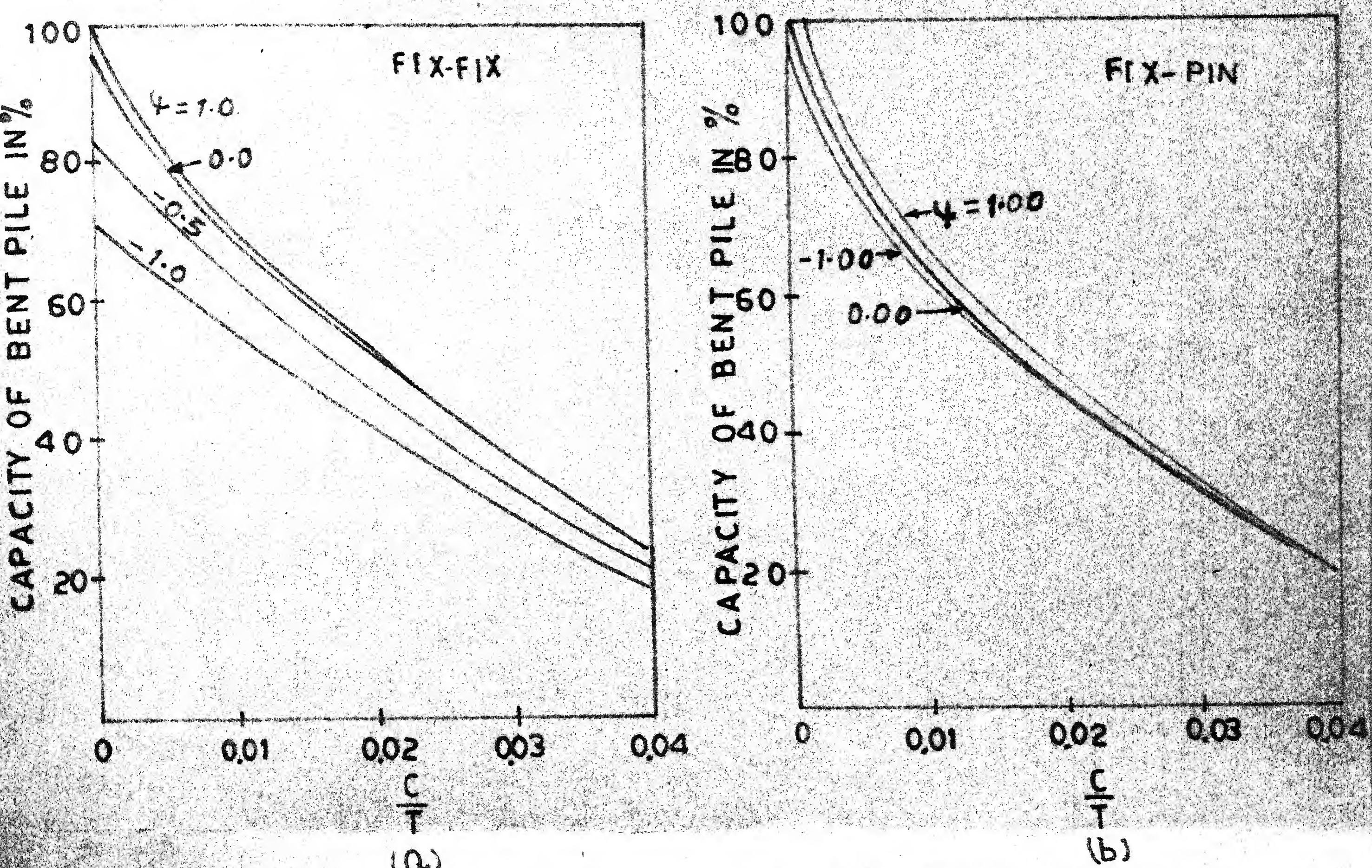
4·6 CAPACITY OF BENT PILE Vs Z_m IN COHESIVE SOILS



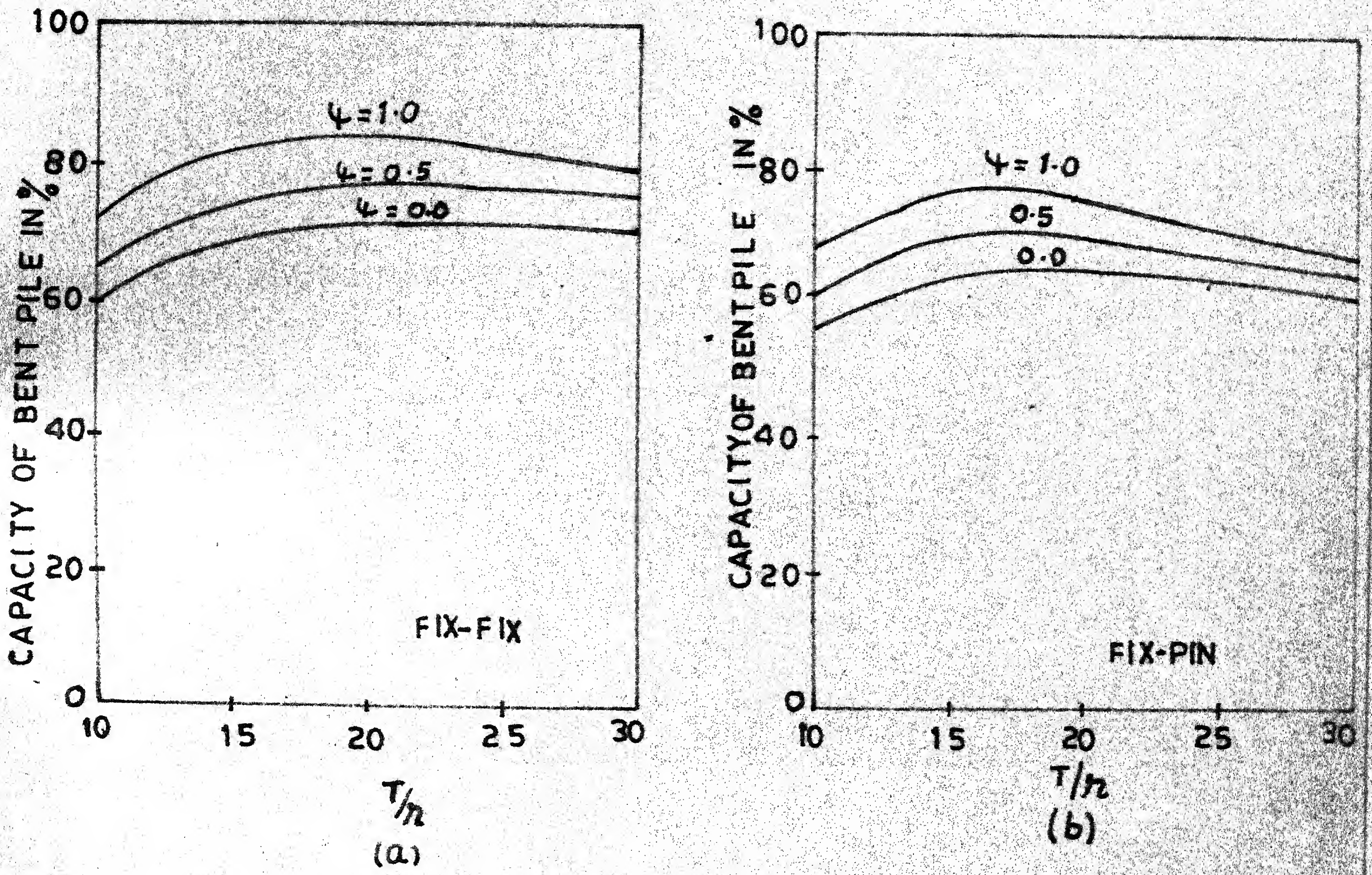
4·7 CAPACITY OF BENT PILE Vs Z_m IN COHESIONLESS SOILS



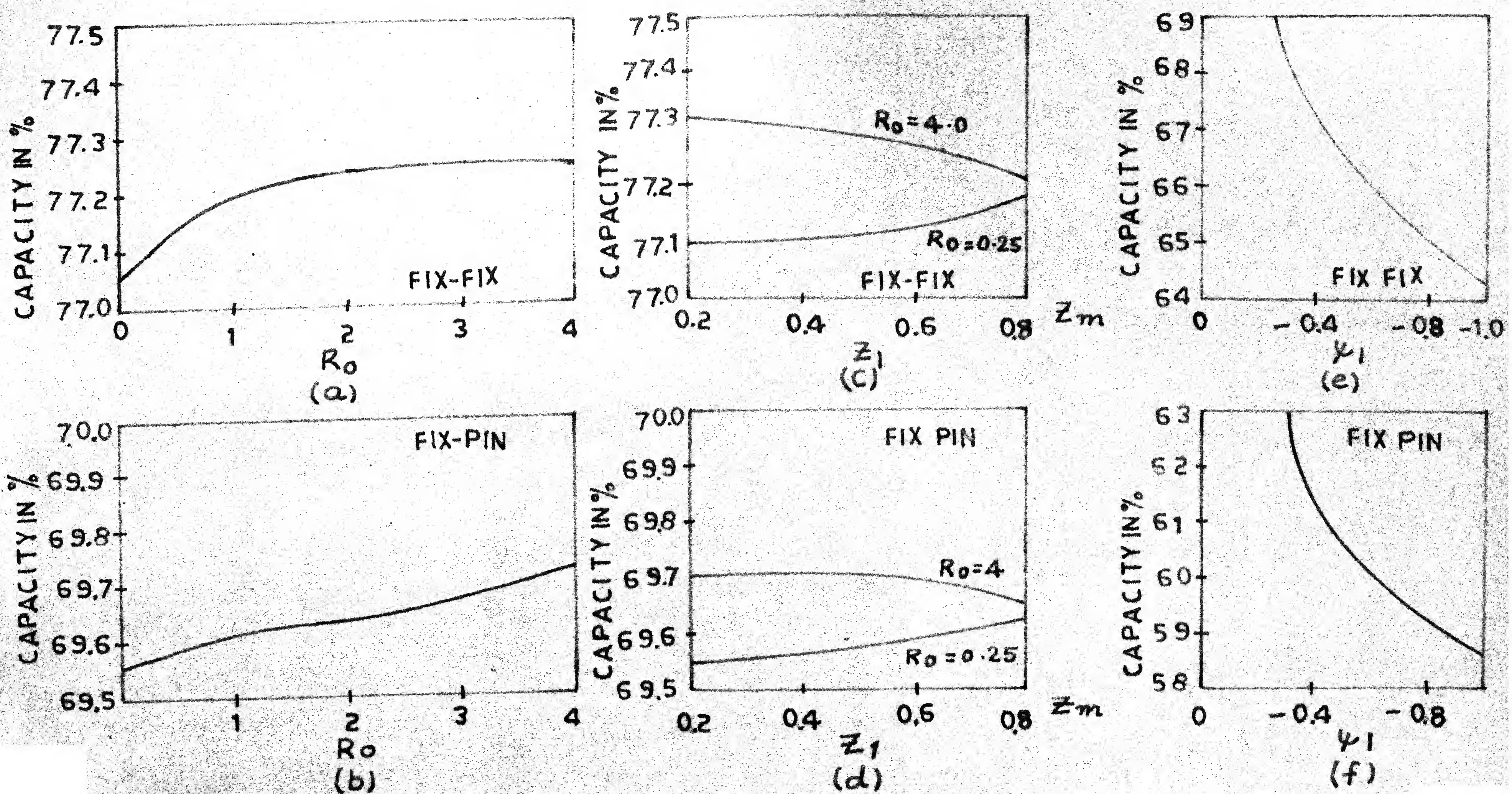
4·8 CAPACITY OF BENT PILE Vs C/T IN COHESIVE SOILS



4.9 CAPACITY OF BENT PILE Vs C/T IN COHESIONLESS SOILS



4·10 CAPACITY OF BENT PILE Vs T/R IN COHESIVE SOILS



4·11 CAPACITY OF BENT PILE Vs R_0, Z_1 AND ψ_1 IN COHESIVE SOILS

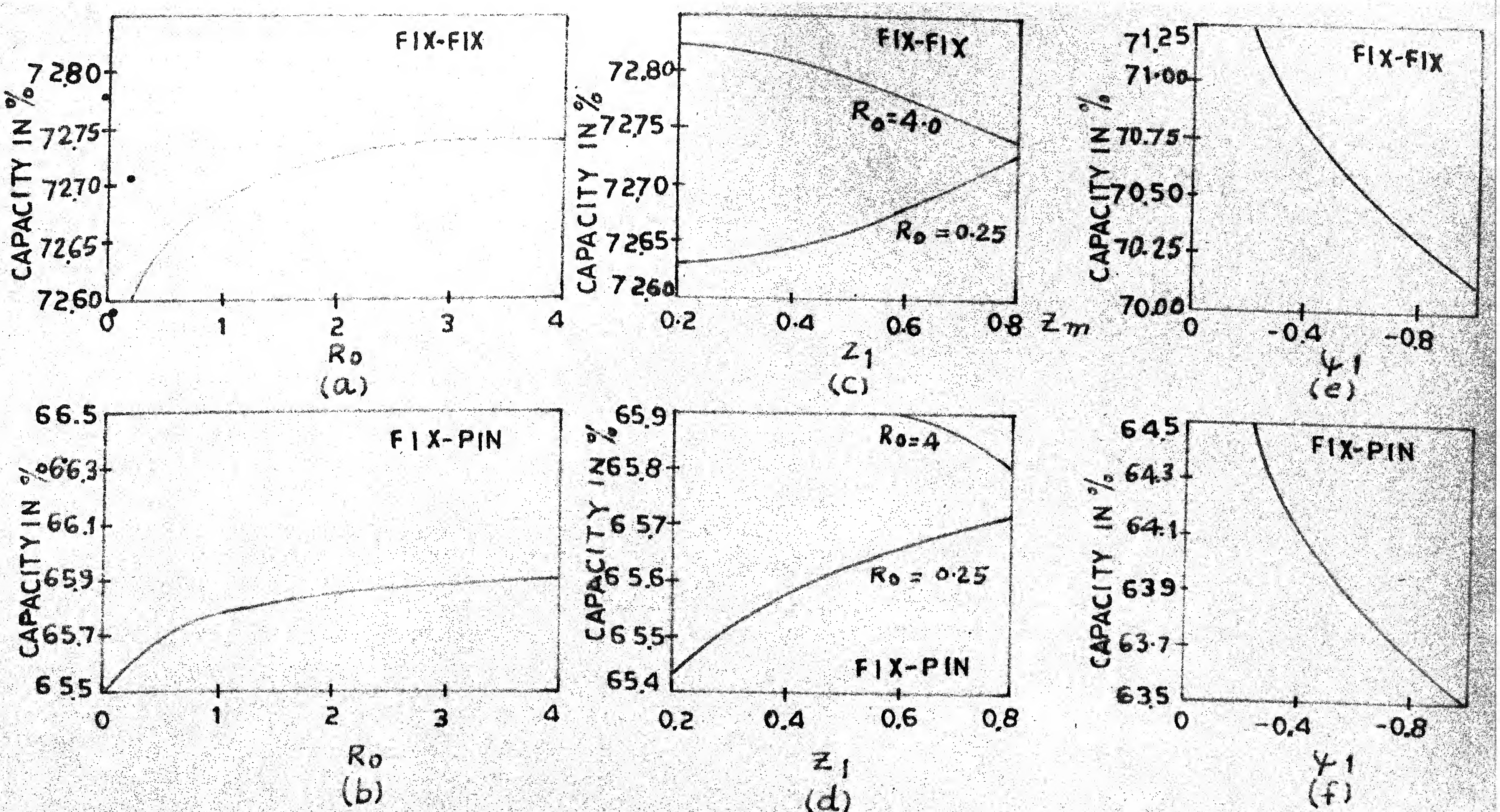
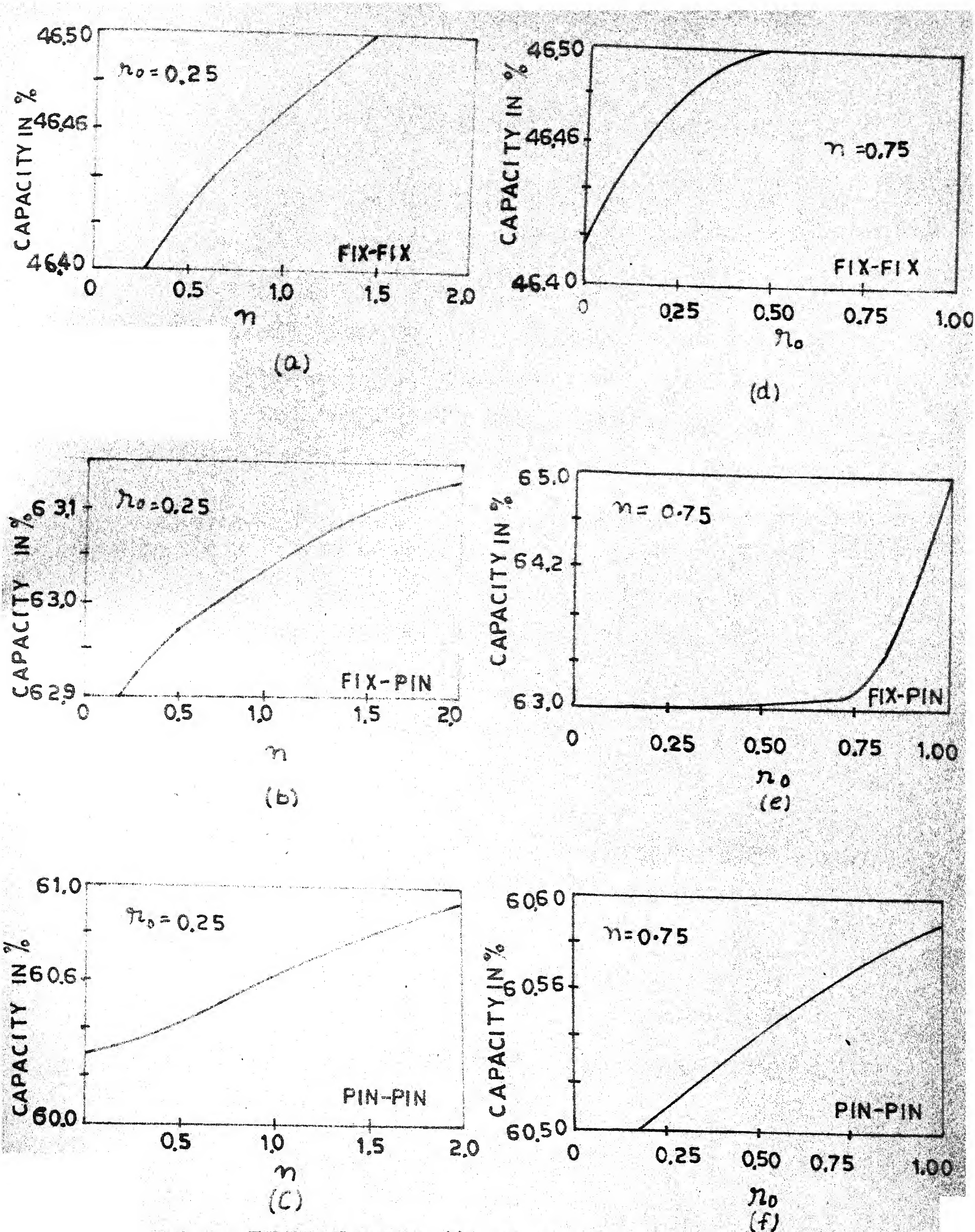


FIG.4-12 CAPACITY OF BENT PILE Vs R_o, Z_1 AND ψ_1 IN COHESIONLESS SOIL



4.13 EFFECT OF POLYNOMIAL VARIATION OF SOIL MODULUS ON CAPACITIES

CHAPTER V

SUMMARY AND CONCLUSIONS

It is desirable to drive the piles straight, however, it is seldom possible in practice to drive a pile straight without deviations from the vertical axis. Many investigators have contributed to the analysis of deformed piles in homogeneous soils which are briefly reviewed in Chapter II.

In this investigation an attempt is made to study the effect of layered soils on the load carrying capacity of bent steel pipe pile taking into consideration important governing parameters such as boundary conditions, initial shape, offset, soil modulus etc.

Polynomial expressions satisfying natural and geometric boundary conditions are selected arbitrarily for fix-fix, fix-pin and pin-pin end conditions. The soil is idealized by the Winkler medium and the layer coefficient is used to define the ratio of the coefficient of subgrade reactions of each layer.

The initial stresses in the pile are evaluated from the assumed initial shape using general elastic-flexure equation. The beam column equation is used to obtain the axial load response of the bent pile.

For the case of two layered soil system having constant soil modulus in each layer a closed form solution is obtained using two boundary conditions at each end of the pile and four compatibility conditions at the interface of the two layers. The maximum bending stress in the pile is obtained by adding the bending stress due to additional deflection caused by the imposed load to the initial bending stresses developed due to deformed shape of the pile. The load carrying capacity of straight pile is determined from the column concept as recommended by AISC Code of Practice. The load carrying capacity of the bent pile is obtained using the interaction formula given by AISC for columns subjected to both compressive and bending stresses. The highest axial load which causes the axial and bending stresses along the length of bent pile satisfying interaction formula is determined by iteration process and is expressed as a percentage of the capacity of straight end bearing pile.

All the linear parameters are non-dimensionalized by a relative stiffness factor which accounts the rigidity of the pile and the soil modulus. In the column concept of pile design the capacity of straight pile depends on the slenderness ratio of the pile. To study the effect of slenderness ratio on the capacity of bent pile, a slenderness parameter is defined as the ratio of relative stiffness of the pile to the radius of gyration of the pile.

The load carrying capacities of bent piles with different initial shapes for each set of end conditions are determined and the initial shape of the pile is found to be most important parameter which determines the capacity of bent pile.

The beam column equation is suitably modified to obtain the axial load response of bent piles with side friction in cohesive and cohesionless layered soils. In cohesive soils the coefficient of subgrade reaction is assumed to be constant with depth and the variation of axial load in the pile is assumed to be linear. In cohesionless soils the coefficient of subgrade reaction is assumed to be linearly varying with depth and the variation of axial load in the pile is assumed to be parabolic. To account for any deviations from soil having constant and linear variation of soil modulus with depth, the effect of polynomial variation of soil modulus with parabolic variation of axial load in the pile is also studied, on the load carrying capacity of initially bent piles. Solutions to the initially bent friction piles are obtained using finite difference technique.

The following conclusions can be drawn from the parametric study conducted on initially bent steel pipe pile in this investigation.

<u>End Bearing Piles:</u>

1. For the initially bent piles the load carrying capacities calculated from soil failure criteria are much higher than the capacities calculated from structural failure criteria. Thus the bent pile design is governed by the structural considerations.

2. In the layered soil additional deflections and additional bending moments due to axial load decreases as the strength of the bottom layer increases. The load carrying capacity of a bent pile increases as the strength of bottom layer increases. In case the bottom layer is weaker than top layer the load carrying capacity of bent pile increases with the increase in the thickness of top layer. When the bottom layer is stronger than the top layer load carrying capacity decreases with the increase in the thickness of top layer.

3. As the load on the initially bent pile increases the additional deflections and additional bending moments increases. In homogeneous soils, for fix-fix case, the maximum additional deflections occur simultaneously at a depth of 1/4th and 3/4th pile length. For fix-pin and pin-pin end conditions the maximum additional deflections occur at a depth of 3/4th pile length. For fix-fix pile the maximum additional bending moments occur simultaneously at the ends of the pile and at depths of 1/4 and 3/4 pile length. For fix-pin pile

maximum additional bending moment occurs simultaneously at top end of the pile and at a depth of 3/4th pile length. For pin-pin pile the maximum additional bending moments occur at a depth of 3/4th pile length.

4. As the initial offset increases the initial bending moments, additional deflections and additional bending moments due to applied load increases for all the end conditions. The load carrying capacity of bent pile reduces with the increase in the initial pile offset for all the end conditions. For the smaller offsets the maximum percentage load carrying capacity is obtained with slenderness parameters between 15 and 20. At other slenderness parameters the capacity will be smaller. However, at larger offsets the percentage load carrying capacity of highly slender piles is large.

5. At constant offset the load carrying capacity of a bent pile increases with the increase in length of the pile upto a certain length after which there is no significant increase in the capacity with the increase in length of the pile.

6. The load carrying capacity of bent piles is primarily a function of initial shape. Irrespective of initial pile offset the percentage load carrying capacity of bent pile increases as the radius of curvature of the initial shape increases. For the same radius of curvature the percentage

load carrying capacity is maximum for fix-fix end conditions and minimum for pin-pin end conditions.

Friction Piles:

1. The soil friction significantly reduces the additional deflections and additional bending moments. For the same tip resistance the reduction in additional deflections and bending moments due to applied load is more in cohesive soils than in cohesionless soils. The additional deflections and bending moments are increased due to negative skin friction. The percentage load carrying capacity of a bent friction pile is more than that of an end bearing pile. The load carrying capacity of a bent pile with negative skin friction is less than that of an end bearing pile.

2. The effect of friction is more significant for longer piles than for shorter ones.

3. The effect of friction is more significant at smaller pile offsets than at larger pile offsets

4. The load carrying capacity of a bent friction pile is insensitive to the strength of bottom layer. Similarly the polynomial variation of soil modulus has insignificant effect on the load carrying capacity of bent pile.

Suggestions for Further Research:

From this study several new ideas have emerged and a few of them are listed below which can be persued further.

1. Analysis of bent piles taking torsional stresses into account.

2. Analysis of bent piles subjected to axial load, lateral load and moment.

3. Analysis of pile groups with some piles bent.

4. Drivability of bent piles.

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